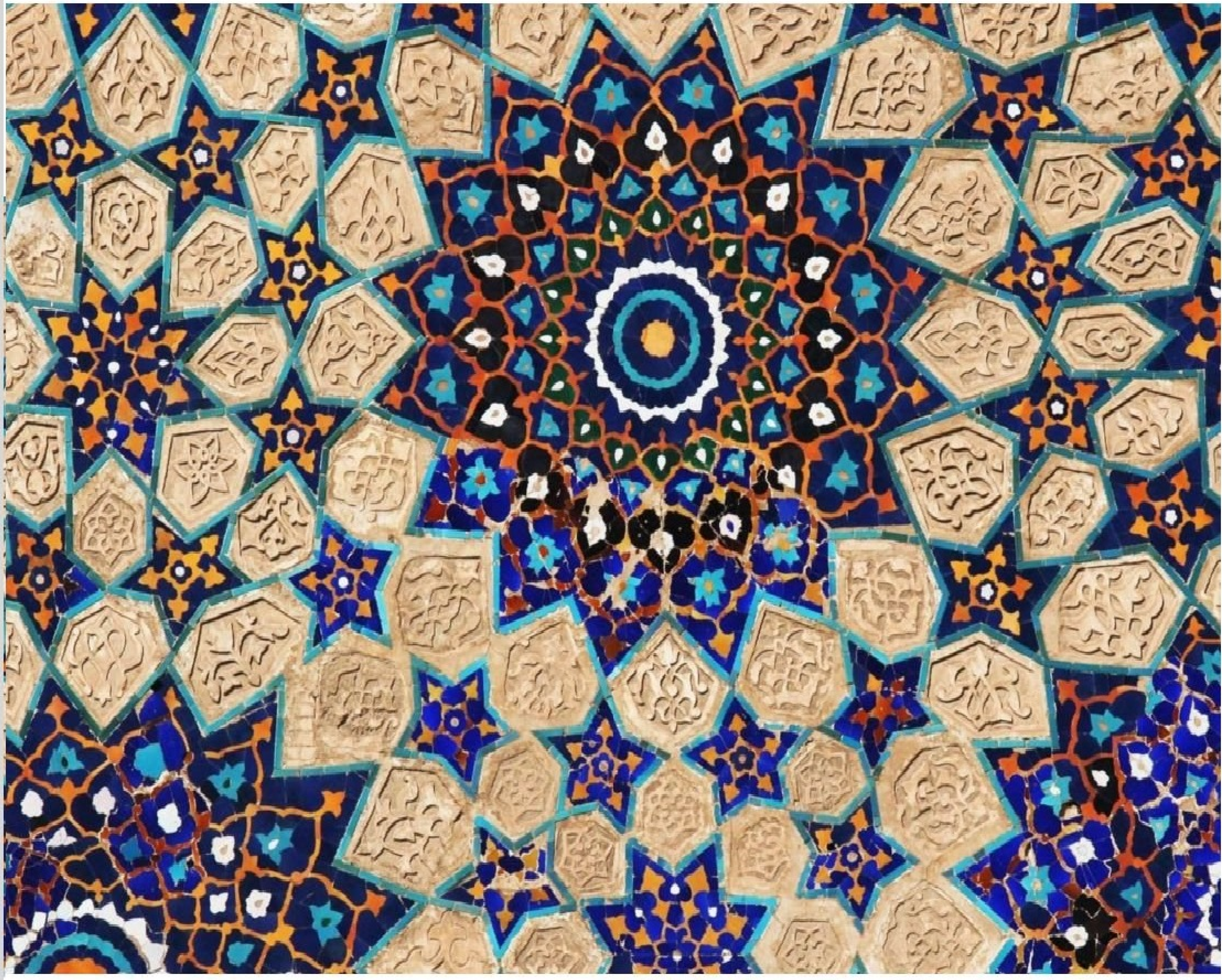

SADRATNAMALA
OF
SANKARA VARMAN

Kerala School of Mathematics, Sanskrit

WITH ENGLISH TRANSLATION AND NOTES
BY
DR. S. MADHAVAN

PUBLISHED BY
THE KUPPUSWAMI SASTRI RESEARCH INSTITUTE
CHENNAI

SADRATNAMĀLĀ

OF

ŚAṄKARAVARMAN

(TEXT ON INDIAN ASTRONOMY AND MATHEMATICS)

WITH ENGLISH TRANSLATION AND NOTES

BY

Dr. S. MADHAVAN

THE KUPPUSWAMI SASTRI RESEARCH INSTITUTE
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FOREWORD

The present work *Sadratnamālā* pertains to a class of mathematico - astronomical works written by scholars of yore in present-day Kerala, known earlier as Malayālam (meaning 'hillocks and valleys') roughly dated from the middle of the 14th Century to the middle of the 19th Century. Most of them were written in Sanskrit and a few in old Malayālam language. To list these works with considerable mathematical content we have: *Tantrasaṅgraha, Karaṇapaddhati, Kriyākarmakarī, Yuktibhāṣā* preceding the present work. All the above works are available in some printed form or other, though not readily comprehensible to readers not knowing Sanskrit or those knowing Sanskrit but not knowing traditional astronomy and enumeration system like *Kaṭapayādi.*

The text of *Sadratnamālā,* on the other hand, was not available in printed form even with these limitations till recently. The late Dr. K. V. Sarma, who was an active bibliographical researcher and who brought out in print a number of works written by Kerala scholars, published a printed version of this work from his collected material prepared quite some years ago. The preparation of this collected material was, infact, under the auspices of a project of the Indian National Science Academy (INSA), New Delhi co-ordinated by me as a member of staff in the Ramanujan Institute for Advanced Study in Mathematics, University of Madras duirng 1988 - 91.

The force behind this project was the late Mr. S. Hariharan, who was Zonal Manager in the Life Insurance Corporation of India and an Actuary by profession. Hariharan used to meet me and discuss with me the contribution of Indian scholars to mathematics earlier to the renaissance in Europe. I could infer from his discussion that he had quite a good scholarship in traditional astronomy and the contributions of Kerala scholars of the medieval period in updating and innovating the Aryabhatan system. Having been part of a team of collaborators, though a late comer, of the late Prof. C.T. Rajagopal who had done significant work relating to exposure of contributions of Kerala scholars of the medieval period to mathematical analysis and approximation (information about which was first recorded by a British Civil Servant C.M. Whish in the Proceedings of the Royal Asiatic Society in 1835), I immediately approved of putting up a project for printing a collated version of the *Sadratnamālā* with English translation and commentary as suggested by Hariharan. On his suggestion, the late Dr. K.V. Sarma and Dr. P. Gopalakrishnan Nambi, Professor of Physics in a college in Kozhikode, were taken as resource persons and the INSA readily provided the funds for the project under my co-ordination.

Dr. Sarma took care of the collating of the text, while Hariharan and Nambi were in charge of translation and technical interpretation. As the other two resource persons pointed out to me, the main thrust of the latter aspect was borne by Hariharan, in spite of his commitments in his office. When three of the six chapters were ready, Hariharan

was affected by serious illness and the work had to stop. Hariharan was slowly recovering, but could not resume work, though he tried his best to regain his original health.

In the meanwhile, INSA closed the project and I could not find a competent successor to Hariharan for five to six years. I was wondering whether anyone would take up a thankless job for the love of it without any financial support. It was during this time that I met Dr. S. Madhavan, who had retired from the University College, Thiruvananthapuram, in a seminar on ancient Indian contributions to science at Anna University, Chennai. I found Dr. Madhavan quite competent to continue the work and he was also ready to take up the completion of the work for the love of it. He has done really a wonderful job critically bringing out the limitations in the author's treatment and the shortcomings in the author's unfinished auto-commentary in Malayalam.

Dr. V. Kameswari of the Kuppuswami Sastri Research Institute (KSRI) was ready to bring out the work as a publication of the Institute and the INSA was generous enough to provide funds for publication through the good offices of Dr. S. Sriramachari, one of its past Presidents.

A unique feature about the *Sadratnamālā* is a procedure given only here for extraction of square roots and cube roots of natural numbers. It is somewhat like the bisection method of present day approximation theory but differs from it significantly in the sense of ignoring remainders. My professional colleague Dr.V.K. Krishnan of Thrissur has given a mathematical justification for the

procedure which is provided in Appendix III of this publication. I thank him for his effort which enhances the evaluation of the author's mathematical skill.

I am very thankful to Dr. Madhavan for completing the English translation and providing technical comments and Appendices to help the readers. I am thankful to the late Dr. K.V. Sarma and Dr. P. Gopalakrishnan Nambi for their earlier participation in translating the work. I end up dedicating the work to the late S. Hariharan who was the real force behind the attempt to make this work available to scholars at large and thanking the KSRI, more particularly Dr. V. Kameswari and the INSA for making this publication possible.

M.S. Rangachari
Formerly Director & Head
The Ramanujan Institute for
Advanced study in Mathematics,
University of Madras

17.12.2010
Chennai

PREFACE

The Institute takes pride in bringing out this publication "*Sadratnamālā* of Śaṅkaravarman", a text on Indian Astronomy and Mathematics, with English translation and notes by Dr.S.Madhavan. With this publication, the Institute is renewing its activities in the field of Jyotiṣa śāstra after a long gap.

Earlier, the Institute had brought out the editions of three smaller texts on Jyotiṣa *viz.*, (1) the *Cintāmaṇi sāraṇikā* of Daśabala of the West Indian school (brought out in 1952), (2) the *Grahacāranibandhana* of Haridatta, a basic text of the Parahita system of astronomy prevalent in South India (brought out in 1954); and (3) the *Grahaṇāṣṭaka* of Parameśvara, a short manual on eclipse (brought out in 1961). Prof.D.D.Kosambi edited the former and the latter two were edited by Dr.K.V.Sarma. All these three texts were brought out at first as supplements to the *Journal of Oriental Research* of the respective years and later were published by the Institute as separate monographs.

In addition, the Institute has also brought out the following two important publications on Astronomy:

(i) The *Vākyakaraṇa* with the commentary *Laghuprakāśikā* of Sundararāja, critically edited with

Introduction, Appendices etc. by Prof.T.S.Kuppanna Sastri and Dr.K.V.Sarma in the year 1962. The *Vākyakaraṇa* is the source-book of the *Vākyapañcāṅga*, the almanac of the Tamil speaking areas of South India. It is a manual (*karaṇa*) in which *vākyas* as sentences and phrases are used as mnemonics for the numbers in the tables. It is intended for practical use with the aim of easy computation.

(ii) *Sanskrit Astronomical Tables in England* by David Pingree of Brown University, USA. It provides the data collected by Prof. Pingree on Jyotiṣa works and authors studied by him at different libraries in England, specifically the British Museum, the India Office Library, the Welcome Historical Medieval Library and the Royal Asiatic Society in London; at the University Library and the Trinity College Library at Cambridge; and at the Bodleian Library at Oxford.

Besides this, the Institute also carried out inter-disciplinary research with the grants provided by the Indian National Science Academy, New Delhi. Dr.A.S.Ramanathan (Retd. Deputy Director-General of Meteorology, Govt. of India) worked on a project on ā*Weather Science in Ancient India*ä which was subsequently published as a book (Rajasthan Patrika Limited, Jaipur, 1993). Dr.K.V.Sarma (Retd. Director, V.V.R.I., Hoshiarpur) worked on "Critical study of the *Bṛhatsamhitā* with the hitherto unpublished commentary *Utpala Parimala*".

Another project on "*Eclipses in Hindu Life and Thought*" was undertaken by Dr.(Mrs.) Jayasree Hariharan in Sept. 1988 under the aegis of Dr.G.Srinivasamurti Foundation, Madras. It is a public charitable trust started in 1982 with the aim to sponsor under its banner projects, publications and so on related to our traditional science and *Śāstras.* When the project on Eclipses was completed and submitted by Dr.Jayasree under the guidance of Dr.S.S.Janaki, the then Director of the Institute, the G.Srinivasamurti Foundation gave permission to Dr.S.S.Janaki to bring out the same as a publication of the Institute, and accordingly the book was brought out by the Institute in 1995. The book has brought together and analysed relevant mythological, semi-scientific, and scientific information on eclipses scattered in Vedic literature, Epics, Purāṇas, Jyotiṣa and Dharma Śāstra literature. The documented study highlighted the fact that ancient Indian astronomers like Brahmagupta, Bhāskara and Śrīpati scientifically computed eclipses. The Appendix of the book carries valuable material from *Atharvapariśiṣṭa* and *Bṛhatsamhitā* on the effects of the actual occurrences of the eclipses in various stages, on human life. Dr.Jayasree herself tried to continue her study on the material provided in the Appendix but could not do so due to her sudden demise.

Lectures and seminars have also been carried out by the Institute from time to time as "Science and Sanskrit" in general and Jyotiṣa in particular. The most important of these is the seminar on Jyotiṣa conducted during the

Kuppuswami Sastri Birth Centenary celebrations (1980-81). Dr.Arka Somayaji of Tirupati presided and the following modern and traditional scholars participated in the seminar: Dr.George Abraham, Sri K.V.Seshadrinathan, Sri L.Narayanan, and Dr.N.Gangadharan, all of Madras (now Chennai); Dr.K.V.Sarma of Hoshiarpur, Mr.M.A.Bhatt of Tirupati; Sri K.V.Narayanan of Bangalore, Sri H.K.Krishnamurthi of Mysore and Sri Krishna Bhatt of Manipal. While the emphasis was on astronomy, the subjects expounded being Vasiṣṭha Siddhānta – formulae for determining the motion of the moon, concept of Rāhu, errors in observation of planets, astronomical study in Kerala and astronomical data in the Purāṇas; astrological matter was also discussed like limitation of astrology and astrological influence of Mars. The president hailed it as the first seminar of its kind.

Again, in Oct. 1994 a day long symposium on "Sanskrit and Science" was held by the Institute in connection with its Golden Jubilee celebrations. The participants were traditional scholars as well as computer scientists, drawn from different organisations like Matscience and Department of Sanskrit. University of Madras, I.I.T. Madras, Birla Institute of Technology, Pilani, CDAC, Pune; Department of Computer Science and Automation and Indian Institute of Science, Bangalore. Of the papers presented, the one by Dr.V.Krishnamurthy, Former Deputy Director and Professor of Mathematics, BITS, Pilani on the "*Clock of the Night Sky*" dealt with the 27 formulae related to

nakṣatras that help one to fix the time of night by looking at the position of stars in the sky. He later developed this into a book of the same title.

In connection with the Platinum Jubilee of the Samskrita Academy, Madras, the Institute along with the Samskrita Academy conducted a one-day symposium on "Ancient Indian Scientific Knowledge" on 25 Feb. 2003, in which Dr.V.Krishnamurthy of BITS, Pilani and Prof.M.S.Rangachari, formerly Director of Ramanujan Institute of Mathematics, University of Madras participated and spoke on Indian Mathematical Tradition.

The Institute has also been turning its attention to Jyotiṣa field in connection with M.Phil and Ph.D. theses. Dr.Sita Sundar Ram was awarded Ph.D. degree by the University of Madras for her dissertation entitled "Bījapallava of Kṛṣṇadaivajña: A Critical Study" *Bījapallava* is the commentary on *Bījagaṇita* of Bhāskara II and has been considered quite a valuable contribution to the field of Algebra as it carries *upapattis* (proofs). This was critically evaluated by the scholar and was highly commended by the examiners.

Following this, the dissertation on "Doṣa Parihāras in Bṛhad Parāśara Horā Śāstra" by Mr.K.Srikkanth, was submitted to the University of Madras and was awarded the M.Phil degree. Presently he is working on "Critical study of *Pāṭīgaṇita* of Śrīdhara and *Gaṇita Tilaka* of Śrīpati" for his Ph.D. Recently a newly enrolled scholar has been

advised to work on "Surds" in Indian mathematical tradition.

In this continued research in the field of Jyotiṣa, the publication of the text "*Sadratnamālā*" of Śaṅkaravarman with English Translation and notes by Dr.S.Madhavan is another feat achieved by the Institute. The "*Sadratnamālā*" is a text on Astronomy and Mathematics written by a Prince of Kerala by name Śaṅkaravarman belonging to the Kadatanadu Royal lineage.

The text was taken up for publication on the advice of Prof.M.S.Rangachari (Former Director, Ramanujan Institute of Advance Study in Mathematics, University of Madras) who was involved with the work from its edition onwards. As could be seen from the For^word by him, the text has had some hurdles to reach this final shape as given by Dr.S.Madhavan. When all the help has been rendered by Prof.M.S.Rangachari including going through the press copy (and later proofs) and after making improvements in the content as well as the presentation, it was a job already partly completed and made easy for the Institute to publish it. The Institute is greatly indebted to him for all his help.

In addition to all these he has been instrumental in securing the financial aid from the Indian National Science Academy for the publication of this book, through the good offices of Dr.S.Sriramachari who was one of the past Presidents of INSA. We are thankful to both of them for their instantaneous and generous help.

We are also thankful to the authorities of INSA, New Delhi for granting funds for publishing this book. It has been of great help to the Institute which is a non-governmental organization and is standing on its own legs with sporadic financial support of philanthropists from all over the globe.

We cannot adequately thank Dr.S.Madhavan who took over the responsibilities of his senior colleagues who had worked on the edition and translation of the text. It has only been a love of labour, as far as he is concerned. An erudite scholar in Sanskrit language and literature, it has been quite enlivening for us at the KSRI to work with him on this publication whenever his guidance was required. The introduction to the book written by him which is quite interesting and informative, stands testimony to his scholarship. The Institute is deeply beholden to him for his great service to the cause of indological research.

Mr.K.Srikkanth, Ms.S.Anusha, Mrs.V.Yamuna Devi, Mrs.V.Uma Maheswari (Ph.D scholars of the Institute) and Mr.S.N.Krishna, Mrs.B.Ramadevi and Mrs.R.Subasri (Research assistants of the Institute) headed by Dr.(Mrs.) Sita Sundar Ram assisted the editorial committee in this publication.

Special thanks are due to Mr.B.Ganapathy Subramanian (Madras Sanskrit College, Chennai) and Ms.K.Vidyuta (Post-graduate student in Sanskrit) for computerising the entire text, formatting it, making

necessary alterations and corrections wherever needed, with patience and interest and to Mrs.M.Srividhya of the Institute who joined the team at the final stage. Mr.B.Ganapathy Subramanian is also to be congratulated for the nice cover design of the book.

Our thanks are due to M/s.Vignesha Printers for the neat printing and nice get-up.

30.12.2010 **V.Kameswari**
Chennai Director

INTRODUCTION

Nothing is as exciting as the study of the Universe with its vast expanse, the celestial peregrinators that baffle mankind with their movements and the wheels within wheels that mystify their motion. In every early civilization, the study of planets and stars stand out pre-eminently. In the Vedic civilization also it had a fundamental role though the exact extent to which it developed is yet to be assessed. Nevertheless, the legacy from the Vedic civilization was great and it triggered the study of the subject in all details that led to the substantial contributions to the subject in the later periods.

1. Indian Astronomical Tradition

The Indian tradition of Jyotiṣa refers to the eighteen propounders, *viz.,* Sūrya, Pitāmaha, Vyāsa, Vasiṣṭha, Atri, Parāśara, Kaśyapa, Nārada, Garga, Marīci, Manu, Aṅgiras, Romaka, Pauliśa, Cyavana, Yavana, Bhṛgu, and Śaunaka. The original works of these sages are practically not extant, though parts remain scattered in fragmentary forms. Except the work *Vedāṅga Jyotiṣa*[1] we do not have any standard work relating to the astronomy of Vedic times.

1. *Vedāṅga Jyotiṣa* of Lagaḍa, with Translation and Notes by T.S. Kuppanna Sastry, Indian National Science Academy, New Delhi, 1984.

The Vedic Astronomy inspired the later astronomers as evidenced by the statement of Varāhamihira (*Bṛhatsamhitā* I.2):

prathamamunikathitam avitatham
avalokya granthavistarasyārtham |
nātilaghuvipularacanābhiḥ
udyataḥ spaṣṭam abhidhātum ||

This means that Varāhamihira after going through the elaborate and infallible treatises of the early sages thoroughly, attempted to present the contents of them, in a form which is neither large nor short.

The galaxy of astronomers of India includes the following persons:

(1)	Āryabhaṭa	(5th/6th Century A.D.)
(2)	Varāhamihira	(6th Century A.D.)
(3)	Brahmagupta	(6th-7th Century A.D.)
(4)	Haridatta	(6th-7th Century A.D.)
(5)	Lalla	(8th-9th Century A.D.)
(6)	Govindasvāmin	(8th-9th Century A.D.)
(7)	Śaṅkaranārāyaṇa	(8th-9th Century A.D.)
(8)	Vaṭeśvara	(10th Century A.D.)

(9)	Muñjāla	(10th Century A.D.)
(10)	Śrīpati	(10th Century A.D.)
(11)	Āryabhaṭa II	(10th Century A.D.)
(12)	Bhāskara II	(12th Century A.D.)

and some later Kerala astronomers. There is a work called *Sūryasiddānta* traditionally believed to be given as *upadeśa* by the Sun God to Mayāsura. It is in all probability, a work after Brahmagupta. This *Sūryasiddānta* differs considerably from the one described by Varāhamihira in *Pañcasiddhāntikā*. Varāhamihira also refers to Pradyumna and Vijayanandin in his *Pañcasiddhāntikā*. But their works are not extant.

The above list is not exhaustive. It is estimated that about 100,000 manuscripts in Jyotiṣa are available. If they see the light of day, a clear picture of the achievements of India in astronomy and mathematics will be presented to the world.

2.Kerala school of Astronomy and Mathematics

The school of Astronomy founded by Āryabhaṭa had disciples in Kerala and it was here that the school flourished[2]. Vararuci (4th Century A.D.) gave *Candravākyas* for the computation of the Moon's position and his method held its sway for a long time. Haridatta (C. 650 - 700) propounded the

2. K.V.Sarma, *A History of the Kerala School of Astronomy*, V.V. R. Inst., Hoshiarpur, 1972.

Parahita System of astronomy founded at Tirunavay, in 683 A.D., to rectify the Aryabhatan system which became inaccurate by this time. The '*Bhaṭasamskāra*' or '*Śakābda-samskāra*' was introduced in this connection to rectify the mean positions of planets obtained by the Aryabhatan method.

Govindasvāmin (8th-9th century) who wrote a super-commentary on *Mahābhāskarīya*, Śaṅkaranārayaṇa (C. 850-900 A.D.) who commented on *Laghubhāskarīya* and Sūryadevayajvan (C. 1191-1250) who wrote a commentary on *Āryabhaṭīya* contributed substantially to the field of Astronomy. The stormy development of Mathematics in the post-Bhāskara period in Kerala is really unique in the history of mankind. Mādhava of Saṅgamagrāma who gave the infinite series for R sine, R cosine and circumference of a circle (which *ipso facto* reduces to the infinite series for π) and $\tan^{-1}x$ undoubtedly towers above others. He wrote *Veṇvāroha*[3] which can be interpreted as an application of the properties of periodic functions.

Parameśvara of Vaṭaśreni (1360-1455) introduced the *Dṛk* system and rectified the *Parahita* system, which had become inadequate for astronomical calculations by his time. Nīlakaṇṭha Somayājin (C. 1444-1545) wrote several thought-provoking works which include *Tantrasaṇgraha*, *Golasāra* etc., and conjectured the heliocentric motion of planets. Jyeṣṭadeva (C. 1500-1610) wrote the work *Yuktibhāṣā* to summarise the mathematical and astronomical concepts which prevailed at his

3. S. Madhavan, *Veṇvāroha from Modern Perspective* (to appear); S. Madhavan, "Models in Indian Astronomy", National Seminar on Indian Intellectual Tradition, Sree Sankaracharya University of Sanskrit, Kalady, 2004.

time and provided the rationale of the various concepts and theorems, playing the roles thereby of Euclid who wrote the *Elements* and Ptolemy who wrote *Almagest.* Acyuta Piṣāraṭhi (C. 1550-1621) introduced the correction of reduction to the ecliptic and improved the computation of latitude of the Moon and this was done nearly at the same time by Tycho Brahe in Europe. Putumana Somayājin (C. 1668-1749) who wrote *Karaṇapaddhati* introduced novel methods of computation.

Śaṅkaravarman of Kaṭattanāḍ (C. 1774-1839) who belonged to the intellectual fraternity that actively fostered the growth of Astronomy and Mathematics had at his disposal the works of several master-minds, when he wrote *Sadratnamālā.* The question whether he was influenced by the earlier works will be examined while dealing with the contents of the work (See section 5 below).

3. Śaṅkaravarman: Life and Works

The year of birth of Śaṅkaravarman is given as A.D. 1774 by Govinda Pillai in *Malayālabhāṣā Caritam* (Trivandrum, 1881) and it has been accepted by some scholars[4]. On the other hand, Ulloor S. Parameswara Iyer, Vadakkunkur Raja Raja Varma and S. Venkatasubramonia Iyer[5] take it as A.D. 1801. This is the

4. K.K. Raja, *Contribution of Kerala to Sanskrit literature,* Madras, Second edn. 1980, p. 268; Easwaran Namputhiri, *Sanskrit literature of Kerala,* Trivandrum, 1972, p. 118.

5. Ulloor S. Parameswara Iyer, *Keraliya Sāhitya Caritram,* Vol III, Trivandrum, 1955, p. 499; Vaṭakkumkur Rāja Rāja Varma, *Keralīya Samskṛta Sāhitya Caritram,* Vol IV, Trichur, 1962, p.384; S. Venkaṭasubramonia Iyer, *Kerala Sanskrit literature: A Bibliography,* Trivandrum, 1976, p. 111.

date given in the incomplete edition of the present work of this author (Nadapuram, 1898). K. V. Sarma prefers the date 1774, in view of the fact that the year of composition of the work is Kali 4921 (A.D. 1819) when he would have been only 18 years of age if the year A.D. 1801 is accepted as his year of birth. The year A.D. 1774 certainly sounds more logical.

The perpetration of atrocities by Tipu Sultan (C. 1766-81) made the princes of Malabar seek asylum at Travancore, which was ruled by Rāmavarman. Tipu was defeated at Seringapatam in A.D. 1799 and after that the Britishers held their sway in Malabar. Belonging to one of the royal lines of Malabar, Śaṅkaravarman's family fled the country and took refuge under the Mahārāja Rāmavarma of Travancore. Being only third in the line of princes, Śaṅkaravarman spent his time in scholarly pursuits. He is said to have been attached to Mahārāja Svāti Tirunāl (1813-47) who being a man of profound scholarship and a patron of learning, supported him. During the British control of Malabar, many erstwhile kings encouraged academic excellence and this gave rise to the development of literature, and different branches of learning[6].

Śaṅkaravarman himself mentions that he hailed from the Porlārtiri family, and invokes Goddess Pārvati installed at Lokamalayārkavu (*lokāvanidhara-sarid-ārāma*). His personal deity was Lord Kṛṣṇa installed at a place called Kāray-āṭu (Sanskritised as Kṛṣṇameṣa), as evidenced by the v.51 of Chapter V. He was the third in the line of princes, the first and

6. Dr. K.K.N.Kurup, *History of Tellicherry Factory*, Sandhya Publications, Calicut, 1985.

second being Udayavarman and Rāmavarman as indicated in the following (*Sadratnamālā* I. 3):

śrī porlātirivamśamauktikamaṇeḥ śrīkeralālaṅkṛte-
rāryasyodayavarmaṇaḥ śubhamateḥ śrībhaimibhūmīpateḥ |
śrimatsodararāmavarmayuvarājāryājñaya tanyate
tantram śaṅkaravarmaṇedamakhilajyotirvidām prītaye ||

He describes Udayavarman as *bhaimībhūmī pati*, the king of Ghaṭotkaca *bhūmi*, the Sanskritised form of Kaṭattanāḍ. It appears that Udayavarman was a titular ruler. The work was undertaken by him at the instance of Rāmavarman, the crown prince.

Being a brilliant astronomer, astrologer and poet and one endowed with good command of language he brought out this work, *Sadratnamālā* – 'Garland of Precious Gems'.

4. Contents and influence of earlier works

The work contains 212 verses as suggested by the term *tridaśamuni saṅghāta* in Chapter VI, v. 58 which means the well-formed product of 3,10 and 7 or 210 and indicates the approximate number of verses (or a few more). The term *bhāḍhyā* means increased by 4 or 27 and thus the number of verses can be 214 or 237. It is doubtful whether the text has come down to us in its original form.

Let us examine the contents of the various chapters.

Chapter I deals with the names of decimal numerals and defines the eight operations, viz., addition, subtraction,

multiplication, division, squaring, extraction of the square root, cubing and extraction of the cube root. Apart from the usual method for the extraction of square root and cube root as found in *Līlāvatī*, etc., the author gives a different method which is of considerable intrinsic interest.

Chapter II deals with the different measures of arcs, time, lunar days, planets, stars, almanac, length, weights of grains, monetary units and directions.

In Chapter III definition of the rule of three, the *Kaṭapayādi*[7] system of numerals, the time elements of the almanac, the method of getting the mean sun, the moon, planets, lunar day, *yoga*, and *karaṇa* are given. It also gives the method of getting the

7. The *Kaṭapayādi* system in vogue in the South (see ch.II Notes under v.3)

A vowel not preceded by a consonant has value 0. In a conjunct consonant the value of the last consonant has to be taken. A consonant not followed by a vowel has no value, when followed by a vowel the value is independent of the vowel. Thus *ma, mā, mi, mī* etc., have the same value namely five. In the South Indian version *ḷa* is included to indicate 9. But the value of *ḷa* (cerebral) has to be decided carefully. Thus *aḷiḥ* can mean 30 or 90 because the word which means a bee is written as *aliḥ* in the north and as aḷiḥ in the south. In the scheme of representing the numbers the extreme left indicates the units, the next place shows the tens and so on. Thus the expression *hariḥ sevyaḥ* means 1728. In the *Candravākyas* of Vararuci, *ḷa* is used to indicate 9 and also 3. The 118th *Vākya* is *dhūḷī syadrajño'yam* and it means $10^r\ 21^0\ 39'$ since the number indicating minutes can not exceed 60 and thus *'ḷi'* stands for 3. On the other hand, in the 60th *Vākya*, *digvyāḷo nāsti* which means $8^r\ 19^0\ 06'$, *ḷa* means 9. That the system is typically South Indian indicates that Vararuci was from the South.

time elapsed after sunrise and sunset. The author uses the *Kaṭapayādi* system in general.

Chapter IV deals with *jyās* and arcs. It contains the infinite series for π, R sine, R cosine etc., and many properties of *jyās*. It is exhaustive and it summarizes the knowledge on *jyās* during his time. The influence of *Karaṇapaddhati* of Putumana Somayājin is clearly discernible in this chapter, though the author includes some results not found in that work.

In Chapter V the five elements, *chāyā* (shadow), *vyatīpāta* (the time when the sum of the *sāyana* longitudes of the Sun and the Moon is 180^0 or 360^0 and their declinations are equal), eclipses, *mauḍhya* or combustion, when the planets come close to the Sun in the zodiac and become invisible and *śṛṅgonnati* the elevation of the Moon's horns or a measure of the Moon's

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There are people who believe that the system of *Kaṭapayādi* was introduced by Vararuci. But many important points have to be noted here. *Mahābhārata* is called *Jayā*, which means 18 because it consists of 18 parvans, there are eighteen chapters in the *Bhagavadgītā*, eighteen *akṣauhiṇīs* took part in the war which lasted for eighteen days. This is the traditional interpretation. This shows certainly the antiquity of the system. In the astrological *sūtras* of Jaimini this method is used according to commentators. This work can be placed between 5th century B.C. and first century A.D. using internal evidence. There is a system of numerals associated with *Sāmaveda* which is similar to this. Since *Sāmaveda* is associated with Jaimini, it appears that the *Kaṭapayādi* system was in existence much earlier than Vararuci, though the exact date has to be settled by research. Though Jaimini, who is the author of astrological *sūtras* is not necessarily the author of Pūrvamīmāṃsā, was perhaps in the line of disciples of Jaimini.

phase are discussed. The influence of *Pañcabodha* is clearly perceptible in the chapter. Occasionally he deviates from the method of *Pañcabodha* and gives different methods mainly in the process of successive approximation. Even the parameters he uses are found in *Pañcabodha*, in general.

In Chapter VI he deals with the *parahitagaṇita*, the methods of computation of planetary position etc. The various parameters and the methods in *Karaṇapaddhati* are found. The methods of finding the divisors of different kinds, *kuttākārakriyā*, and its applications, and the methods of forming various tables for month, year etc., are all akin to the methods in *Karaṇapaddhati*.

Surprisingly the *Tantrasaṇgraha* of Nīlakaṇṭha Somayājin which can be regarded as a break-through has influenced Śaṅkaravarman very little. Though Śaṅkaravarman refers to the *śīghroccas* of Mercury and Venus as the real mean positions in his auto-commentary, he does not elaborate the ideas of *Tantrasaṅgraha* in his work. His focus is on *Parahita* system and he mentions some aspects of *Dṛk* system. The author might have planned a longer work and dropped the idea later. *Yuktibhāṣā* written earlier to the present work might have influenced him. But he has not mentioned the rationales of the theories even in the auto-commentary. On the theoretical side like the theory of *jyās*, he has certainly updated his work and also in his reference to tangent, cotangent, secant and cosecant, but in the applications like computation of planetary positions, he sticks to the old theories.

The name *Pañcabodha* is also a bit ambiguous. There are several works with that name and some of them are anonymous. There is a text of the name ascribed to Putumana Somayājin, and another to Puruṣottama. The *Pañcabodha* referred to here is anonymous, written in ten *khaṇḍas* and very commonly used to teach the students.

5.Special features

Books on Astronomy are very often written in a prosaic style, with dry-as-dust technicalities, though there are exceptions like those of Varāhamihira, Bhāskara and others. The author of *Sadratnamālā* is a poet incarcerated in the narrow confines of Astronomy in which his ever roaming fancies do not get any outlet. Astronomy offers little scope for showing his poetic talents. Yet the author transcends the barriers caused by the technical framework and offers a poetic touch to the work. Though astronomers are often content with *Anuṣṭub* metre or *Āryā* metre, the composition of which is simple, Śaṅkaravarman revels in using unusual metres. His model is perhaps Varāhamihira who employs sixty four metres and *daṇḍakas* and indulges in a metrical extravaganza while describing the effects of transits of planets in his *magnum opus,* the *Bṛhatsamhitā.* In the *Bṛhajjātaka* he uses metres even to convey hidden meanings as fully borne out in the commentary *Apūrvārthapradarśikā*[8].

8. A.N. Srinivasaraghava Iyengar, *Apurvārthapradarśikā,* Adyar Library Series, Madras, 1951.

Śaṅkaravarman also enchants his readers with quite a large number of metres. The metres found in this work include *Vasantatilakā, Sragdharā, Viyoginī, Śārdūlavikrīḍita, Āryā, Anuṣṭub, Upacitrā* (a variant of *Mātrāsamaka*), *Śālinī, Dodhaka, Vidyunmālā, Kabarī, Indravajrā, Pramāṇikā, Mātrāsamaka, Aupacchandasika, Sragviṇī, Śikhariṇī, Māṇavaka, Pṛthvī, Praharṣiṇī, Kusumamañjarī, Mālinī* and *Mandākrāntā.*

He employs *Vasantatilakā* often. The section on Candracchāyāgaṇita is couched in *Śragviṇī*, a metre giving the effect of dancing. Perhaps, he composed it on a moon-lit night, with the mind dancing with pleasure and wrote in a manner resembling the following verse of Līlāśuka:

aṅganāmaṅganāmantare mādhavo
mādhavaṃ mādhavaṃ cāntareṇāṅganā |
itthamākalpite maṇḍale madhyagaḥ
sañjagau veṇunā devakīnandanaḥ ||

That he was a devotee of Lord Kṛṣṇa is clear from many places and particularly from stanza Chapter V, v. 51 and the episodes of the Lord naturally influenced him. To describe the *jyās*, he uses *Kusumamañjarī*, the metre used by Nārāyaṇabhaṭṭathiri to describe Lord Kṛṣṇa's *rāsakrīḍā.*

Yatibhaṅga:

There are instances of *yatibhaṅga* or breaking the rules of caesura. In general, great masters of the language do not

indulge in this. But as it has been said '*niraṅkuśāḥ kavayaḥ*' meaning that poets are free, one can accept this. There are 'approved *yatibaṅgas*'. The metre *Natkuṭaka* is an example. It is also called *Kuṭaka, Narkuṭaka* and *Nardaṭaka.* Kedārabhaṭṭa defines it thus: '*yadibhavato najau bhajajalā guru natkuṭakaṁ*'. The commentator Nṛsimha observes that the *yatis* are at 7 and 10. It is also defined thus:

'*hayadaśabhirnajau bhajajalā gītī narkuṭakam* |'

The metre *kokilaka* has the following definition:

'*muniguhakarṇavaiḥ kṛtayatim vada kokilakam* |'.

These two differ only in caesura. In the first they are given by 7 and 10 and in the second by 7, 6, and 4. Often this is used without *yati,* as illustrated by the following (*Campūrāmayaṇa,* Yuddhakāṇḍa, 49) :

atha madagarjitairadhikatarjitadikkaribhi-
rdaśavadanastadā daśadigantaramantarayan |
samaramukhe sakhela padacaṅkramato vidadhe
harikulamākulam jaladhimādivarāha iva ||

In '*samaramukhe sakhela*' the *yati* after the 7th syllable is not observed. This metre is also called *Markaṭaka.* Being '*markaṭaka*' or monkey, perhaps there are no fixed points of jump! (*markaṭasya yatheccham plavanam*!). Piṅgala's *Chandaḥsūtras*[9] defines the metre *Avitatha* thus:

9. Piṅgala, *Chandaḥ Śāstram,* with the commentary of Halāyudha, Chaukhambha, 2002.

avitathamnjaū bhjaūn jlaūg |

This is the same as *Markaṭaka* and no *yati* is prescribed.

Apart from such examples violation of caesura is treated as a defect of the poem (*kāvya doṣa*). There are instances of breaking the rules of caesura (*yati*) in *Sadratnamālā*[10]. For instance in the line (V. 2):

iṣṭaḥ sāyanabhāskaraḥ svacaraliptāḥ svantarābhyam kṛtaḥ |

10. The interesting thing is the association of Svāti Tirunāl with Śaṅkaravarman, who entertained a similar indifference to the rules of Caesura.

Mahārāja Svāti Tirunāl (1813 - 1846), being a scholar in Sanskrit, English, Tamil, Telugu, Kannada, Hindusthani, many other languages and music, contributed in no small measure to Sanskrit literature and Music. Hailed as Dakṣiṇa Bhoja, a title which fits him appropriately, his works display great devotion and scholarship. Svāti Tirunāl had excellent command of language and there are instances when he indulges in uninterrupted flow of expressions, reminiscent of *Naiṣadhīyacarita.* Apart from literary works, he had to his credit several musical compositions and *upākhyānas* in which musical compositions are mixed with Sanskrit verses. One important trait in his writings is his indifference to the rules of caesura as evidenced by the occasional violation of rules. For instance the following verse from his work *Bhaktimañjarī* (2.16) illustrates this:

prāpte tasyāḥ sadaitatateṛhanta ghore' vamāne
kṣoṇīśānām sadasi mahatām īśa douṣṭyādripūnam |
vema vyāpāramapi caturī sparśalesam vinā yā
śriman vāsāmsyatunata jayet sā kṛpa cāturī te ||

The metre is *Mandākrāntā* and the rule for caesura is violated in '*vema vyāpāramapi*'. In the case of Svāti Tirunāl his deep devotion to the Lord, the ineffable bliss he derived at the moments of ecstasy spontaneously manifested in the form of poetry. The rules of caesura are quite insignificant and caused, perhaps, only impediment to effective expression.

this rule is not followed. The metre is *Śārdulavikrīditā* and the caesura occur at 12 and 7. The word *lipta* is broken at the 12th place.

Unusual Metres:

Śaṅkaravarman was a lover of novelty. He has used metres not found in works like *Vṛttaratnākara.* For instance consider the following verse (VI.28) :

yojanarūpo bimbavyāso
vahnimayārkasyodyadbhāvaḥ |
śakalo'bhrūva prāleyāmśor
mṛṇmaya bhūmerātmā nityaḥ ||

Except in the third quarter it is the same metre as the one used in *Bṛhajjātaka* (XI.9) :

karkiṇi lagne :atsthe jīve
candrasitajñairāyaprāptaiḥ |

Yati is like a musical pause. When one recites a verse there can be a pause in places depending on the tune chosen. *Vidyunmālā* is defined in *Vṛttaratnākara* as - *mo mo go go vidyunmālā.* On the other hand, *Śrutabodha* gives the following definition:

sarve varṇa dīrgha yasyām viśramaḥ syād vedair vedaiḥ |
vidvadbṛndair vīṇāvāṇī vyākhyata sa vidyunmālā ||

Here *yati* occurs after the fourth syllable in each quarter, whereas no *yati* is mentioned in earlier definition. If one feels that there is a musical pause there can be *yati.* Svāti Tirunāl was a great scholar in music and he might have had reasons to justify his violation of the rules of caesura. As in the case of *Vidyunmālā*, if *yatis* can be developed in different ways and the musical manifestation is enjoyable there is no harm in dropping or changing the places of *yati.* With the association of Svāti Tirunāl, Śaṇkaravarman also might have shared his views.

meṣagate'rke jātam vidyāt
vikramayuktam pṛthvīnātham ||

This is formed by breaking the second long syllable of *Vidyunmālā* into two short syllables. He could have used the expression '*śākalamasti*' instead of '*śakalobhrūva*' and retained the same metre in all the four quarters. Further, *abhruvā* is the *uttama puruṣa* dual of *laṅ* of the root *brūñ*. It means 'we two said' meaning the author and earlier writer.

Why is this discord? It may be a deliberate attempt to show his aversion to conventions. The third quarter comes under *Mātrāsamaka* and so the entire verse can be treated as one belonging to *Mātrāsamaka* in which each quarter contains 16 syllabic instants. But there are constraints imposed even on this. This best known metre is defined as *mātrāsamakam navamolgāntam*, in which there are 16 syllabic instants, the ninth syllabic instant is formed by a short syllable and the last by a long syllable. The verse under consideration does not conform to this. One can mix up the different varieties of *Mātrāsamaka*, like *Upacitrā*, *Viślokā*, *Vānavasikā* etc., and form what is called '*pādākulaka*'. But the verse of Śaṅkaravarman does not come under this. One does not understand the need for this experimentation with metres.

Combinations:

Upajāti metre is well-known and it is generally a combination of *Indravajrā* and *Upendravajrā*, bearing names *Kīrti, Vāṇī, Mālā, Śālā* etc., and there are 16 'types' including *Indravajrā* and *Upendravajrā*. One can combine other metres also, *Indravamśa* and *Vamśastha* for instance, as evidenced by the following verse of Māgha (*Śiśupālavadha,* XII. 3):

hastasthitākhaṇḍitacakraśālinam
dvijendrakāntam śritavakṣasam śriyā |
satyānuraktam narakasya jiṣṇavo
guṇairnṛpāḥ śārṅgiṇamanvayāsiṣuḥ ||

Śaṅkaravarman also uses such a combination, but he employs *Indravamśa* in one quarter and *Vamśastha* in three quarters in the following (II.3) :

pratatparā ṣaṣṭiguṇā hi tatparā
viliptikā saivamasau tathākalā |
saivam lavastattridaśāhatirbhaved
rāśiḥ sa mārtāṇḍaguṇo bhamaṇḍalam ||

On another occasion he uses the metre *Kabarī*, which is not quite common (IV.9) :

jīvārdhakṛteriṣuṇā
labdena yutam tamiṣṭam |
vyāsapramitam paridhe-
riṣtasya vidurgaṇakāḥ ||

In fact the second quarter ends with the word '*iṣtam*'. Since it does not tally with the definition of the metre, the reading has to be changed to *iṣum*. This metre is reminiscent of Varāhamihira's verse *Bṛhajjātaka* (VI.3) :

pāpāvudayāstagatau krūreṇa yutaśca śasī |
dṛṣṭaśca śubhairna yadā mṛtyuśca bhavedacirāt ||

The metres used in this work and their full import form a topic for specialized study. Only a partial survey has been made above.

Poetic Skill:

Though there is not much scope for showing his poetic skill, Śaṅkaravarman introduces fanciful expressions occasionally. He says for instance (V.43) :

mārārivardhita ihāstu murāribhaktaḥ

This means that the figure has to be increased by *mārāri* (225) and then divided by 225 (*murāri*). Incidentally, it refers to a devotee of Lord Viṣṇu (*murāribhakta*) who prospers by the grace of Lord Śiva (*mārāri vardhita*) and thus a paradox is introduced.

There are many places indicative of the author's knowledge of grammar.

Though the special features are not studied exhaustively, it is to indicate that detailed study from the above points of view is desirable.

6. Inaccuracies and shortcomings of the text :

The materials used for the study are (i) the transcript made available by K.V.Sarma, which incorporates the differences in readings and (ii) the text with the auto commentary. There are some places in the text showing inaccuracies, relating to the metre, expression etc. We shall enumerate them one by one. The account on metres includes some of these.

(i) Consider Chapter III, v. 27 : The text reads thus:

raśerarkeṇaiṣyā bhuktā liptā bhaktāḥ svacchede nāḍyaḥ ।
kalyāt paścāt pūrvāḥ ṣaḍbhāḍyārkeṇaivam cāstāt ॥

The intended metre is *Vidyunmālā.* But there is an excess of one syllable in the second quarter and a shortage of one syllable in the third quarter. This can be corrected as follows:

raśerarkenaiṣya bhukta
liptā bhaktāḥ svacchede syuḥ ।
nāḍyaḥ kalyāt paścāt pūrvāḥ
ṣaḍbāḍhyārkeṇaivam cāstāt ॥

The error has occurred obviously in copying.

(ii) Chapter IV, vv. 41, 42, 43 :

These three stanzas are not commented on by the author. The translation has been supplied to v. 41 and v. 42 which do not give the exact results. Some modifications have been made to interpret them. In v. 43, the expressions are not accurate.

The auto-commentary is probably missing.

(iii) Chapter V, v. 26, which reads thus: (first two quarters):

samhārakaghnaphalabhārdhamavāg digākṣam
pitṛyarkṣakālabhaguṇena kṛtādamuṣmāt ।

In the first quarter it is suggested to multiply 1287 by *palāṅgula.* Then in the second quarter it refers to the finding of *mahājyā* of *kālalagna.* But it is necessary to subtract three *rāśis* from *kālalagna* as is the usual practice and as suggested in the commentary. So it is necessary to incorporate the idea of subtracting three *rāśis* in some place. It seems convenient to be replaced '*digākṣaṁ*' by '*tribhona*'.

(iv) We turn our attention to Chapter V, v. 45, which runs thus:

evam çaitat kālalagnāntaralavagalite
mūḍha bhāge sa dṛśyaḥ ॥

The metre is *Sragdharā* as evidenced by other quarters. In the above quarter the metre is violated.

It can be replaced by the following:

evam kālākhyalagnāntaralavagalite
mūḍhabhāge sa dṛśyaḥ ||

(v) The Chapter VI, v. 50 is as follows:

karkyeṇādikamandadoḥphalajakotyā svīyakoṭīphala-
svarṇinyā vihṛtārdha visṭṛtikṛtirmandaśrutistadguṇāt |

This does not convey the sense. One can reconstruct thus:

kheṭasya sphuṭakotīmandaphalayorvargaikyamūlam tato-
tena syad vihṛtārdhavistṛtikṛtirmandaśrutistadguṇāt ||

(vi) The Chapter VI, vv. 45, 47 and 48 do not give satisfactory results. It is also likely that these two verses (47 and 48) are interpolations and the work contains only 210 verses, as interpreted in Chapter VI, v. 58. In the auto-commentary the Chapter IV, vv. 40, 41 and 42 of are not included. Also it breaks abruptly at the middle of the Chapter VI, v. 32.

Apart from these, there are some minor mistakes. The section on special features clearly describes the irregularity in some metres. One cannot expect a work written by a scholar like Śaṅkaravarman to have such shortcomings. These irregularities may be the by-product of copying and recopying often by people who do not have knowledge of prosody, grammar or language. One has certainly to conclude that the original text composed by the author has undergone distortions in the process of copying.

7. Manuscripts used for the Edition of the Text

Prof K.V. Sarma has made the critical edition of this text after collating seven manuscripts which are independent of

each other, none of them being a direct copy of another. A short summary of these manuscripts is given below:

(i) A1 – refers to Ms No. 8322-B of Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum.

(ii) A2 – Ms. No. R. 4448 of the GOML, Chennai, which is a paper transcript.

(iii) A3 – Ms. No. C-2136 of Kerala University Mss. Library, Trivandrum is in palm leaf.

(iv) B1– Ms. No. 628-D (old No.1076) of the Govt. Sanskrit College Library, Tripunithura, Central Kerala, which is in palm leaf.

(v) B2 – Ms. No. 22177 of the Kerala University Mss. Library, is in palm leaf.

(vi) B3 – This text is an incomplete edition, extending to VI.32, issued with commentary in Malayalam script in 1898.

(vii) B4 – Ms. No.67735 (old No. 21-B-6) of the Adyar Library and Research Centre, Chennai is in palm leaf and is in Grantha script.

More details regarding each of these manuscripts are given in the Introduction by Prof. K.V. Sarma in his critical edition of this text.

8. Method of Translation

In general, literal translation is not done. The method of free rendering is adopted. In some places, the stanzas are incomplete without the commentary. In such cases, explanatory translations are given.

9. Acknowledgement

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Lokahāryadhunīsuniṣkuṭamahībhṛdvamaśamuktaphala-
śrīmacchaṅkara varmaṇa viracitā Sadratnamālāmalā |
nānāvṛttamaṇidyutirvaraguṇā hauṇīmayacchāyayā
samyuktā bhuvi bobhavītu rucirā vidvajjanānām mude ||

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S. Madhavan

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APPENDICES

SADRATNAMĀLĀ
OF
ŚAṄKARAVARMAN

TEXT ON INDIAN ASTRONOMY AND MATHEMATICS

WITH
ENGLISH TRANSLATION

CRITICALLY EDITED TEXT

OF

SADRATNAMĀLĀ

।। शङ्करवर्ममहाराजविरचिता ।।

।। सद्रत्नमाला ।।

अथ प्रथमं परिकर्माष्टकप्रकरणम्

(मङ्गलाचरणम्)

श्रीपार्वत्याश्च लोकावनिधरसरिदाराम[1]सान्निध्यवत्याः
पादाम्भोजं गुरूणामपि सततमनुस्मृत्य नत्वारुणादीन् ।
ज्योतिश्शास्त्राब्धिकीर्णां विशद[2] गणित[3]सारोक्ति सद्रत्नमालां
संगृह्यैनां[4] लिखामः पिपठिषुजनसन्धारणा[5]स्वल्पयत्नाम् ।।१ ।।

निरुपाधिकृपातिधीनुपासे क्षितिपीयूषभुजो महानुभावान् ।
यदनुग्रहतो विधूतदोषः परिपूर्णाखिलमङ्गलो भवेयम् ।।२ ।।

Mss. used : A_1 – Ker. Univ. 8322-B ; A_2 – GOML, Madras, R.4448 ; A_3 – Ker. Univ. C. 2136 ; B_1 – Skt. Col. Tripunithura, 628-D ; B_2 – Ker. Uni. 22177 ; B_3 – *Kavanodayam*, Nadapuram (1898) ; B_4 – Adyar 67735.

1. A_1 सरिदतागार, A_3 महितागार
2. B_2 विमल
3. B_1 भणित
4. A_3 संक्षिप्यैनां
5. A_3 सन्धारणां

(ग्रन्थनिर्माणे प्रोत्साहनम्)

श्रीपोळ्ळतिरिवंशमौक्तिकमणेः श्रीकेरलालङ्कृते[1] –
रार्यस्योदयवर्मणः शुभमतेः[2] श्रीभैमिभूमीपतेः ।
श्रीमत्सोदररामवर्मयुवराजार्याज्ञया तन्यते
तन्त्रं शङ्करवर्मणेदमखिल[3] ज्योतिर्विदां प्रीतये ।।३ ।।[4]

क्वेदमतिनिगूढार्थं ज्योतिश्शास्त्रं क्व चाहमलसमतिः ।
श्रीगुरुचरणाम्बुरुहस्मरणं किं किं न साधयति ।।४ ।।[5]

(संख्यास्थानानि)

एकं दश शतं चाथ सहस्रमयुतं क्रमात् ।
नियुतं प्रयुतं कोटिरर्बुदं वृन्दमप्यथ ।।५ ।।

खर्वो निखर्वश्च महापद्मः शङ्कुश्च वारिधिः ।
अन्त्यं मध्यं परार्धं च संख्या दशगुणोत्तराः ।।६ ।।

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1. A_3 श्रीशङ्करालङ्कृतेः
2. $B_{2,3.}$ शुभतेः
3. $B_{2.}$ मखिलं
4. B_1 Omits the verse.
5. $A_{1.2.3}$ add the undermentioned verse here. The commentary does not comment on it. Possibily this is a later addition to the text :

श्रीमद्वराहमिहिरार्यभटादभीष्टः शब्दार्थरीतिविरहः शुभदोऽत्र नूनम् ।
श्रीकालिदास–सुकुमारकवी स्वकाव्यमाङ्गल्यतः परिणतिं महितां गतौ हि ।।

(परिकर्माणि)

संख्यानां युतिवियुती गुणनं हरणं च वर्गमूले च ।
घनघनमूले चैतत् साध्यतमं गणितमाहुराचार्याः ।।७।।

(सङ्कलितव्यवकलिते)

यथास्थानं व्युत्क्रमेण क्रमेण यदि वाङ्कयोः ।
मेलनं युतिमत्राहुर्वियुतिं च वियोजनम् ।।८।।

(गुणकर्म)

गुण्यान्त्योपान्त्यादीन[1] सर्वान् गुणयेद् गुणेन पृथगङ्कान्[2] ।
गुण्यान् गुणखण्डसमान् खण्डैस्तैर्वाथ तद्युतिर्गुणनम्[3] ।।९।।

(भागहरणम्)

यद्घ्नो हारो हार्यसमस्तत्फलं हरणे भवेत् ।
हार्याद् धृतिः स्वानधिकहारकेण तथोत्क्रमात् ।।१०।।

(वर्गपरिकर्म)

तुल्योभयहतिर्वर्ग एकतः[4] क्रमशः पदैः ।
का वा धेनुस्तटे शुभ्रा तुङ्गो धावेद् वृषो यदि ।।११।।

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1. $B_{2,3.}$ गुणान्त्येपान्त्यदीन्
2. $B_{2,3.}$ पृथगशङ्कान्
3. $B_{2,3.}$ गुणम्
4. A_2 reads एकशः ; B reads स एकात् क्रमशः ; B_4 omits एकतः

स्थाप्योऽन्त्यवर्गः शेषोऽपि द्विघ्नान्त्यघ्नो निजोपरि ।
उपान्त्यादिम् अथो[1]त्सार्य भूयोऽप्येवं क्रिया कृतिः ।।१२।।

[2]खण्डद्वयहतिर्द्विघ्नी खण्डद्विकृतियुत् कृतिः ।
यद्वाभीष्टोनाढ्यवधोऽभीष्ट वर्गयुता[3] कृतिः[4] ।।१३।।

(वर्गमूलम्)

शुद्धवर्गस्य मूलेन द्विघ्नेनावर्गतो हतम् ।
तदादिमूलं तद्वर्गः शोध्यो वर्गात् पुनस्तथा ।।१४।।

(घनपरिकर्म)

ज्या हृत् सीरी भाति शरण्यस्तत्पुरि गूढाङ्गः श्रीकृष्णः ।
धीरोऽसावेकादिनवान्तं तुल्यत्र्यभ्यासोऽत्र घनः स्यात् ।।१५।।[5]

घनेऽथ तन्मूलवर्गतदादि त्रिवधे ततः ।
आदिवर्गान्त्यत्रिवधे युतेष्वङ्केष्वथो घनः ।।१६।।

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1. $B_{2,3}$ उपान्त्यादीनथो
2. B_2 omits the verse.
3. $B_{1,2,3,4}$ युतः
4. A_3 कृती
5. B mss. have an alternative verse for the present verse:
   यज्ञे जह्नुः सारो भीतिघ्नः शारिका चिकुरः ।
   गूढाङ्गः श्रीकृष्णो धीरोऽसावेकतः समत्रिवधः ।।
   ($B_{2,3}$ शूद्रिका for शारिका ; $B_{2,3}$ समत्रिविधः)

द्विधाकृतात्मात्मघातस्त्रिघ्नस्तद्विघनान्वितः ।
इष्टोनाढ्यात्मात्मघातो वात्मनिघ्नेष्टवर्गयुक् ।।१७।।

(घनमूलम्)

घनमूलस्य वर्गेण त्रिघ्नेनाघनतोऽन्त्यतः ।
लब्धस्य वर्गस्त्रयादिघ्नः[1] शोध्याश्चाद्याद् घनाद् घनः ।।१८।।

इष्टाप्तेष्टैक्यार्धमिष्टमविशिष्टं कृतेः[2] पदम् ।
घनमूलं द्विराप्तेष्टयोगार्धमविशेषितम् ।।१९।।

इति शङ्करवर्मनिर्मितायां सद्रत्नमालायां प्रथमं प्रकरणम्[3] ।।

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1. $A_{3.}$ त्र्यन्त्यघ्नः
2. $B_{2,3.}$ मवशिष्टं कृतैः
3. $A_{1,2,3.}$ इति सद्रत्नमालायां प्रथमप्रकरणम् ।
   $B_{1.}$ इति सद्रत्नमालायां परिकर्माष्टकप्रकरणम् ।
   $B_{4.}$ इति सद्रत्नमालायां परिकर्माष्टकप्रकरणं प्रथमम् ।
   $B_{2.}$ No colophon.

अथ द्वितीयं परिभाषाप्रकरणम्

(कालमानम्)

[1]गुर्वक्षरं विघटिका घटिका[2] दिनं च
पूर्वाणि षष्टिगुणितानि निजोत्तराणि ।
त्रिंशद्गुणं दिवसमत्र च माससंज्ञं[3]
मासोदिवाकरगुणः खलु सावनाब्दः ।।१ ।।

पर्येत्यजस्रं भगणो यत्प्राणैः खखषड्घनैः ।
चक्रलिप्ताश्च तत्संख्याः षट् प्राणा भविनाडिकाः ।।२ ।।

(कलादिपरिभाषा)

प्रतत्परा षष्टिगुणा हि तत्परा
विलिप्तिका सैवमसौ तथा कला ।
सैवं लवस्तत्त्रिदशाहतिर्भवेद्
राशिः स मार्ताण्डगुणो भमण्डलम् ।।३ ।।

सप्तविंशतिभं चक्रं राशिः सांघ्र्यृक्षर्युग्मकः[4] ।
राशौ नक्षत्रदिङ्नाड्यः शतं त्रिंशच्च पञ्च च ।।४ ।।

1. A_3 Marginal note : द्वितीयप्रकरणप्रारम्भः ।
2. B_4 Hapl. om. of घटिका
3. A_3 , B_{1-3} संज्ञः
4. B_3 युग्मगः

(तिथिस्वरूपम्)

चन्द्रार्कान्तरचक्रे तिथयस्त्रिंशंद् यतस्ततो राशिः ।
सार्धतिथिद्वययुक्तस्तत्र हि नाड्यः शतं सपञ्चाशत् ।।५ ।।[1]

(ग्रहाः नक्षत्राणि च)

ग्रहाः सूर्येन्दुभौमज्ञगुर्वच्छार्क्यहिकेतवः ।
मेषादयो राशयः स्युरश्विन्याद्याश्च तारकाः ।।६ ।।

(पञ्चाङ्गपरिभाषा)

वासराः सूर्यवाराद्यास्तिथयः प्रतिपन्मुखाः ।
करणं कृमिसिंहाद्यं[2] योगा विष्कम्भपूर्वकाः[3] ।।७ ।।

त्रिंशच्चतुश्चतुः कल्पा मान्दिनाड्योऽर्कवासरात् ।
स्वपञ्चमोदिता रात्रावेताः सूक्ष्मा निरक्षभे ।।८ ।।

अहनि प्राह्ण[4] पूर्वाह्ण[5] मध्याह्ना अपराह्णकः[6] ।
सायाह्नश्चेति कालाः स्युर्व्यक्षे षड्घटिकामिताः ।।९ ।।

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1. A_2 transfers this after verse 13 in this chapter
2. A_3 सिंहाद्याः
3. $B_{2,3}$ योगो विष्कम्भपूर्वकः
4. B_3 प्राह्ण
5. B_3 पूर्वाह्ण
6. B_3 अपराह्णकः

(दैर्घ्यमानम्)

योजनाष्टसहस्रांशो दण्डस्तच्चरण: कर: ।
तज्जिनांशोऽङ्गुलं तस्य षष्ट्यंशो व्यङ्गुलं स्मृतम्[1] ।।१० ।।

(धान्यादिमानम्)

क्रमाच्चुतुर्गुणं मानं कुडुब: प्रस्थ आढक: ।
द्रोणो वह: खारिकेति घनहस्तमितावधि:[2] ।।११ ।।

(गुरुत्वमानम्)

तुलाशतांश: पलमेतदङ्घ्रि:[3] कर्षोऽस्य माषोऽवनिपालभाग: ।
गुञ्जास्य पञ्चांशक एतदर्धं यवो हि गुञ्जात्रितयं तु वल्ल: ।।१२ ।।

(मुद्राणां परिभाषा)

निष्कस्य षोडशांश: स्याद् द्रम्मोऽस्यांशस्तथा पण: ।
पणस्याङ्घ्रि: काकणी तद्विंशत्यंशो वराटक:[4] ।।१३ ।।

1. $B_{1,2,3}$ transfer this after the next verse. A2 transfers this after II.13

2. A_3, $B_{1,4}$ तावधि

3. B_1 मेकमंघ्रि:

4. A_2 वराटिका

(दिग्योनय:)

प्रागादियोनय इह ध्वजधूमसिंहा[1]
विष्टिर्वृष: खर इभो बलिभुक् क्रमेण ।
त्रिघ्नाष्टभक्तपरिशिष्टगृहादिनाह-
हस्ताङ्गुला[2]न्यतरजास्त्वयुजोत्र शस्ता : ।।१४।।

इति शङ्करवर्मनिर्मितायां सद्रत्नमालायां
द्वितीयं प्रकरणम्[3] ।।

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1. $A_{2,3}$ सिंह
2. $A_{2,3}$ हस्ताङ्गुलोऽन्य
3. $A_{1,2}$ do not end the chapter here, but continue it with the next, the verse numbers also being continuous.

   A_3 , B_4 इति सद्रत्नमालायां द्वितीयप्रकरणम् (B_4 द्वितीयं)

अथ तृतीयं पञ्चाङ्गप्रकरणम्

(देवतानमस्कारः)

मातङ्गास्यं भारतीं कृष्णमैशं मार्ताण्डादीनानताः स्मो गुरूंश्च[1] ।
[2]श्रीलोकाम्बां दक्षिणामूर्तिमेषां गीर्नः श्रेयोऽनुग्रहोत्था ददातु ।।१ ।।

(त्रैराशिकम्)

त्रैराशिकं प्रमाणेच्छाफलैरिच्छाफलाहतिः ।
प्रमाणेन फलेच्छाभ्यासत इच्छाफलं हृतम्[3] ।।२ ।।

(कटपयादिसङ्ख्यानियमः)

नञावचश्च शून्यानि संख्याः कटपयादयः ।
मिश्रे तूपान्त्यहल् संख्या न च चिन्त्यो हलस्वरः[4] ।।३ ।।

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1. B_1 गुरुं च
2. A_1 omits the latter half of the verse.
3. A_3 भवेत् for हृतम् । ($A_{1,2,3}$ have an additional verse here :
   हारं प्रमाणं सर्वत्र गुणमिच्छा तदन्वये ।
   गुणप्रमाणस्य फलं लब्धिरिच्छाफलं वधात् ।।)
4. $A_{1,2,3}$ have this verse before verse I.6 while two extra verses, which have not been commented upon in the commentary, occur here with serial verse numbers :
   ग्राह्याविष्टेन तष्टावपि[1] गुणकहराविष्टनिघ्नौ च तौ वा
   शेषोऽन्योन्याहतौ यस्तदुपहृतगुणच्छेदकौ चानुपाते ।
   किं चोपेक्ष्यो गुणो वा भवति गुणहतो हारको हारकश्चेत्
   छेदोच्छिन्नो गुणश्चेद् गुण इह गणकैर्हारकश्चाप्युपेक्ष्यः ।।

(कल्यब्दानयनम्)

क्षीराब्धिगाढ्य[1] वर्षान्तकोलम्बाब्दा: कले: समा: ।
धीसौख्य गाढ्यशाकाब्दा वा मध्यार्कभ्रमोद्भवा: ।।४।।

कल्यब्दनिघ्नमुकुटोल्बणकृष्णतालं
गुर्वक्षरादिभयलेशदनक्रहीनम् ।
कल्यादितो दिनगणोऽच्छदिनाद् द्वयोने
सप्ताप्तशेष इनसंक्रमणध्रुवोऽस्मिन् ।।५।।

(रविमध्यम:)

दिनं स्वसंख्यानाड्यूनं तिथ्यर्धांशघटीयुतम् ।
श्रेष्ठाहि[2] रत्नादियुक्तं[3] वृषत: सूर्यमध्यम: ।।६।।

(इष्टदिनाहर्गणम्)

इष्टोदयार्कमध्याढ्यगतकल्यब्दसञ्चय: ।
धीजगन्नूपूरक्षुण्णस्तत्समाप्तोऽप्यहर्गण:[4] ।।७।।

फलैकदेशोस्तु गुणो गुणघ्नात् प्रमाणराशे: फलसंहृतं हृत् ।
अत्राधिकोनघ्न[2] फलांशभक्तं हृद्घ्न[3] प्रमाणात् क्षयवृद्धिहार: ।।

1. A_3 भक्तावपि, 2. A_3 त्रैराशिकोनघ्न, 3. A_3 हृद्घ्नात्

1. $B_{2,3}$ क्षीराब्धिर्गाढ्य
2. B_1 श्रेष्ठादि
3. B_4 युतं
4. $B_{1,2,3,4}$ ० प्तो ह्यहर्गण:

(रविस्फुटानयनम्)

वृद्धोत्तुंगसुसौख्यहीनकलितो माहात्म्यपाठं त्यजेत्
तच्छेषाद्धटिकादिरत्नकृतपाटीं पीलुनिम्बार्थिनीम् ।
तुष्यन्मातलिमप्यजादिपठितं वाक्यं च नाड्यादिकं
शिष्टे यातभसंयुते दिनकरो योग्यादिसंस्कारतः ॥८॥

योग्यादिनाड्यो ह्यष्टाष्टदिनानां शिष्टवासरात् ।
स्ववाक्यघ्नादष्टभक्तं शिष्टवासरनाडिका ॥९॥

पूर्णमासर्क्षतः स्पष्टसंक्रान्तेः शिष्टसंयुतम् ।
इष्टमासि दिनं यज्ञरत्नाद्याढ्योनितं रविः[1] ॥१०॥

(चन्द्रस्फुटानयनम्)

गोपीधवळच्छायाहीनाद् द्युगणात् त्रिस्थलरम्यैर्भक्ते ।
कालानङ्गैरथ देवेन्द्रैः शिष्टा तद्दिनवाक्यगसंख्या ॥११॥

स्वाप्तैर्निघ्नाः क्रमात् स्युर्विविधनिजवृधाराधनः चाटुनृत्यत्[2]-
कैलासानेकपो धीबहुफलविसुखः स्वध्रुवास्तत्पराद्याः ।
तद्योगे पार्वतीशस्मरहरतनुसंयोजिते देशभेद-
क्षुण्णश्चन्द्राल्पभोगो धनमिह समरेखाप्रतीच्यां ध्रुवःस्यात् ॥१२॥

1. B_2 omits this verse inadvertently.

2. A_1, $B_{1,2}$ नृत्यात्

रेखाप्रतीच्याश्रयदेशभेदनिनाडिकाभ्यस्तसिता मृदंशः[1] ।
धनात्मकः पुङ्गवताडितान्त्यफलान्वितः सूनुहतादिजाड्यः ।।१३ ।।

कालनगाप्तफलघ्नतनीयान् दैवभयेन युतस्त्वृणरूपः ।
तद्विवरोद्धृतभूतलदेवः स्वर्णमयो ध्रुवसंस्कृतिहारः ।।१४ ।।

वाक्ये ध्रुवाढ्येऽहर्मानध्रुवसंस्कारसंस्कृते ।
उदयेन्दुस्तदूर्ध्वाधो वाक्यभेददलं गतिः ।।१५ ।।

अस्तचन्द्रगतिर्वाक्ये पूर्वोने सार्धितादियुक् ।
ध्रुवतत्संस्क्रियाष्टांशघस्रमानकृता विधुः ।।१६ ।।

कलीकृता विधुगती रुद्रस्थानविवर्जिता ।
ध्रुवसंस्कारहाराप्ता लिप्ताः स्वर्णं स्वहारवत्[2] ।।१७ ।।

अद्यश्वोऽस्तमयोत्थेन्दुगत्योर्भेदाष्टमांशकम्[3] ।
अस्तेन्दौ स्वं पूर्वगतेराधिक्याद् ऋणमन्यथा ।।१८ ।।

1. $B_{1,2,3}$ शरांशः
2. A_3 लिप्तिः स्वर्णं तु हारवत्
3. $A_{1,2}$, B_2 मांशकः ; A_3 , B_1 माशकाः

(स्फुटतिथिः योगश्च)

सूर्यस्फुटविहीनं यत्स्फुटमिन्दोस्तिथिस्फुटम् ।
सूर्यस्फुटाढ्ये चन्द्रे तु तयोर्योगस्फुटं भवेत् ।।१९ ।।

(नक्षत्रपादाः करणानि च)

नरांगम् अद्भुतं स्नानं[1] नित्यं नीरलयः क्रमात् ।
अधक्षये ज्ञाननिष्ठा निद्रागारे नभश्चरः ।।२० ।।

नानाज्ञानकृदित्येकराशावृक्षाङ्घ्रयो[2] नव ।
तिथ्यर्द्धं करणं चिह्नं श्रेयोदीपो भरो नगः ।।२१ ।।

निद्रालयनृन्नरयौ[3] भूनेत्रगुणादिगुणौ ।
दस्रप्रतिपन्मुखयोः सन्धी भवतश्चरमे ।।२२ ।।

यज्ञरत्नाद्यष्टमांशयुक्तोनोक्तो रवेर्गतिः[4] ।
तदाढ्योने चन्द्रगती गतियोगतदन्तरे ।।२३ ।।

स्फुटकालाद् भूतभाविनीष्टकाल[5] ऋणं धनम् ।
कालान्तरघ्नी स्वगतिः स्फुटे व्यस्तं विलोमगे ।।२४ ।।

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1. B_1 ज्ञानी for स्नानं
2. B_1 वृक्षपदा
3. B_1, B_3 नुन्नरयौ ; B_4 ननृरयौ
4. B_2, B_3 रवेर्गतः (wrong)
5. A_3 भूतभव्यइष्टकाल

क्रमाद् भतिथियोगानां गतगन्तव्यलिप्तिका: ।
गतितच्छेदतद्योगभागाप्ता निजनाडिका: ।।२५ ।।

भस्यांशार्धं रन्ध्रगुणं दिङ्नाड्यो दिग्गुणं तिथे: ।
द्विघ्ना नवाप्तास्ता[1] भांशा: पञ्चाप्तास्तास्तिथेर्लवा: ।।२६ ।।

(उदयात् पूर्वापरराशय:)

राशेरर्केणैष्या भुक्ता लिप्ता भक्ता: स्वच्छेदे नाड्य:[2] ।
कल्यात् पश्चात् पूर्वा: षड्भाढ्यार्केणैवं चास्तात् ।।२७ ।।

इति शङ्करवर्म निर्मितायां सद्रत्नमालायाम्
तृतीयं प्रकरणम् ।।[3]

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1. $A_{1,2}$ नवघ्नास्ता (wrong)
2. $B_{1,2,3}$ स्वच्छेदननाड्य:
3. A_1 इति सद्रत्नमालायां द्वितीयप्रकरणम् ; A_2 इति सद्रत्नमालायां द्वितीय: ; A_3 , B_1 इति सद्रत्नमालायां तृतीयप्रकरणम् ।

अथ चतुर्थं ज्याचापादिप्रकरणम्

(वृत्तपरिधिः)

वर्गाद् व्यासस्यार्कनिघ्नात् पदं यत्
तत्त्र्यंशो यस्ते च तत्तन्नवांशाः ।
द्विघ्नव्येकैकद्विपूर्वौजयुग्म-
च्छिन्नान्यैक्यद्व्यन्तरं वृत्तनाहः ।।१ ।।

व्यासार्कघ्नकृतेः पदेऽग्निभिरतो नीते च तत्तत्फला-
च्चाथैक्याद्ययुगाहृतेषु परिधिर्भेदो युगौजैक्ययोः[1] ।
एवं चात्र परार्धविस्तृतिमहावृत्तस्य नाहोऽक्षरैः
स्याद् भद्राम्बुधिसिद्धजन्मगणितश्रद्धा स्म यद् भूपगीः ।।२ ।।

(महाचापमानम्)

ज्योतिश्चक्रचतुर्भागं खण्डयेद्[2] बहुधा समम् ।
महाधनूंषि तान्यात्मपूर्वचापान्वितानि हि ।।३ ।।[3]

(ज्याचापानयनम्)

चक्रलिप्ताः परार्धघ्नास्तद्वृत्ताप्ताः स्व विस्तृतिः ।
तद्दलं त्रिभजीवा स्यात् साऽर्धितैकभशिञ्जिनी ।।४ ।।

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1. $B_{2,3,4}$ युगोजैक्य
2. $B_{2,3}$ खण्डयन्
3. A_2 transfers this verse after the next, चक्रलिप्ता etc.

द्व्यादिघ्नविस्तृतिदलाप्तधनुर्घ्नचाप-
तत्तत्फलेषु धनुषस्त[1] समान्यधोधः ।
[2]व्यासार्धतश्च विषमाणि निधाय शोध्या-
न्येतान्युपर्युपरि ते भुजकोटिजीवे ।।५ ।।

स्वान्त्यवर्गान्तरपदं कोटिः कोट्यूनसंयुते ।
त्रिजीवे इष्टजीवाया बाणौ बाणाहतिः कृतिः ।।६ ।।

मिथः कोटिघ्नयोस्त्रिज्याहृतयेरिष्टजीवयोः ।
योगभेदौ तयोश्चापयोगविश्लेषयोर्गुणौ ।।७ ।।

आद्यन्तभज्योन्तरमुत्क्रमेण मध्यर्क्षमौव्याथ भुजाभसन्ध्योः ।
समान्तरालांशगुणैक्यभेदाश्चान्त्याद्ययोरुत्क्रमतः क्रमाज्ज्याः ।।८ ।।[3]

जीवार्धकृतेरिषुणा लब्धेन युतं तमिष्टम् ।
व्यासप्रमितिं परिधेरिष्टस्य विदुर्गणकाः ।।९ ।।

1. B_1 धनुषश्च

2. A_1 omits the latter half of the verse.

3. $B_{1,2,3}$ have for this an alternative verse :
स्तेनः[1] श्रीकृष्णः सुगन्धानिलो भद्राङ्गवेपथुः
मग्नाग[2] नरसिंहाज्ञा स्नानाधीना कवेरभूः ।
इष्टचापकृतिघ्नाद्यात् कृत्यान्त्य[3]धनुषो हृतम्
शोध्यं चोपर्यपर्येवमुत्क्रमज्यान्त्यवाक्यजम् ।।
1. B_1 तेन 2. B_1 मग्नाङ्ग 3. B_1 त्रिज्यान्त्य

कोटीहतत्रिगुणबाहुवधे च तस्मात्
तत्तत्फलाच्च भुजवर्गहतात्तु कोट्याः ।
कृत्या हतेषु च धराग्निशराधिभक्ते-
ष्वोजैक्यतस्त्यजतु युग्मयुतिं धनुस्तत् ।।१० ।।

धनुः स्वघनषष्ठांशात् त्रिज्याकृत्याऽप्तवर्जितम् ।
स्वल्पं गुणायतेऽल्पज्या त्वसकृत् तद्युता धनुः ।।११ ।।

एकादिनिघ्नाद् दशभिर्विभक्ताद् व्यासार्धवर्गाद् घनमूलमूनम् ।
विलिप्तिका वा गुणकेन तेन समाल्पजीवा तु युता धनुः स्यात् ।।१२ ।।

(केन्द्रं पदव्यवस्था च)

मध्यस्फुटादिकेन्द्राख्यमुच्चपातादिवर्जितम् ।
त्रिभषड्भायनाद्याढ्यं स्वयं वा दलितं क्वचित् ।।१३ ।।

मेषाद्यमाद्यं भगणार्धमुत्तरं ज्ञेयं द्वितीयं च तुलादि दक्षिणम् ।
क्रमेण मेषादितुलादिदिग्भवान्यर्णं धनं व्यस्तविधौ विपर्ययात् ।।१४ ।।

ओजयुग्मौजयुग्माख्या राशिचक्राङ्घ्रयः क्रमात् ।
भुजाकोटिभुजाकोटिसंज्ञा राशित्रयात्मकाः[1] ।।१५ ।।

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1. $B_{2,3}$ त्रयात्मकः

केन्द्रे त्र्यनधिके तद्दोस्त्र्यधिके षड्भशोधितम् ।
षड्भोनं चक्रसंशुद्धं नवभानधिकाधिके ।।१६ ।।[1]

(ज्याग्रहणं चापीकरणं च)

ऊनाधिकासन्नधनुर्धनुर्भिदा हृद्व्यासभक्तेन धनुर्गुणात् क्रमात् ।
कोटीयुतोना द्विगुणा ततश्छिदा लब्धोनयुग्ज्या भवतीष्टशिञ्जिनी ।।१७ ।।

समीपतज्ज्ययोर्भिदा हृतः स्वकोटियोगतः ।
हरोऽमुना भविस्तृतेहिता तयोर्धनुर्भिदा ।।१८ ।।

(परमक्रान्तिः)

प्राहुः परमक्रान्तिं नित्यां जिनभागानां जीवां प्राञ्चः ।
ह्रासादद्य परीक्षादृष्टाद् रदलिप्तोनानामित्येके ।।१९ ।।[2]

(इष्टक्रान्तिः)

अन्त्यक्रान्तिगुणाभीष्टज्या त्रिगुणाप्ता स्यादिष्टक्रान्तिः ।
इष्टापक्रमकोटिर्द्युज्या त्रिज्याशुद्धा सापमबाणः ।।२० ।।[3]

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1. $B_{1,2,3}$ have an additional verse here :
कोटिचापं विजानीयाद् भुजा चापोनभत्रयम् ।
साध्यतेऽभीष्टचापज्या महाचापगुणैरपि ।।

2. $B_{1,2,3}$ have an alternate verse :
क्रान्तिज्या कृष्णसल्लापैर्गळमर्मघ्नदोर्गुणात् ।
क्रान्तिकोटिद्युजीवा सा त्रिज्याशुद्धोपमाशुगः ।।

3. $B_{1,2,3}$ do not have this verse.

(प्राणकलान्तरम्)

ज्याकोटिघातात् त्रिज्याप्तात् परापमशराहतात् ।
द्युज्याप्तोऽसुकलाभेदः स्वर्णं युग्मौजपादतः ॥२१॥

(क्षेत्रादिकरणम्)

त्रिभुजं च चतुर्भुजं श्रुतिभ्यां स्फुटवृत्तं खलकर्कटेन साध्यम् ।
अथ उर्ध्वमपीह लम्बसूत्रात् समभूर्निश्चलवारिपूरतुल्या ॥२२॥

(शङ्कुः दक्षिणोत्तरानयनं च)

द्व्यङ्गुलव्यासमूलाग्रः समवृत्तऋजुर्गुरुः ।
वितस्त्युच्चः शङ्कुरग्रकेन्द्रसूच्या करोच्छ्रितः ॥२३॥

अल्पवृत्तस्वमध्यस्थशङ्क्वाभाग्रयुतिद्वयात् ।
कृतवृत्तद्वयग्रासमध्यज्या दक्षिणोत्तरा ॥२४॥

(अयनांशः)

शकुन्ताहृत[1] राश्यादिभुजाक्रान्तिधनुर्लवाः ।
इष्टकल्यब्दतः स्वर्णं दिशा परहितायनम् ॥२५॥[2]

1. A_3 शकुन्तहृत

2. $B_{1,2,3,4}$ transfer this verse to the end of the chapter.

अनन्तभक्तो राश्यादिः कल्यब्दात् तद्भुजोद्भवाः ।
राशिज्या दृश्ययनांशा[1] एका[2] धन्या स्थिरा हिताः ।।२६ ।।[3]

(पलाङ्गुलम्)

गोलान्तस्थितसायनभास्वन् मध्याह्नेऽर्काङ्गुलमितशङ्कोः ।
छाया सांशाङ्गुलकप्रमिता स्वा पलभा स्यात् सात्र हरिः श्रीः ।।२७ ।।

(अक्षः लम्बश्च)

शङ्क्वक्षभावर्गयोगमूलेन गुणरम्यभात् ।
लम्बकः पलभाक्षुण्णत्रिज्यातोऽक्षगुणो हृतः ।।२८ ।।

(चरानयनादि)

अक्षगुणत्रिगुणात् त्रिभजीवावर्गादपि निजलम्बकभक्तौ ।
स्वौ गुणहारौ गुणहतदोर्ज्या कोट्याप्ता चरमपमशरासात् ।।२९ ।।[4]

पलभाघ्नापमार्कांशो भूज्या तत् त्रिगुणाहृतेः ।
चरार्धज्या द्युजीवाप्ता चरप्राणो हि तद्धनुः ।।३० ।।

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1. $B_{1,2,3,4}$ read for the quarter as राशिज्यायनभागो
2. $B_{2,3}$ शून्या for एका
3. $B_{1,2,3,4}$ transfer this verse to the end of the chapter.
4. B_2 does not have this verse.

(लम्बज्यानयनम्)

त्रिज्यावर्गाक्षभाकर्णघातात् त्रिज्याप्तवर्जिताः ।
स्वाहोरात्रार्धजीवाप्ता जीवाप्ता हारमौर्विकाः ।।३१।।

त्रिज्याप्तलम्बहतचन्द्रलयस्य वर्ग-
स्त्रिज्याकृतिः श्रित इहाद्युभयघ्नकोट्या ।
हीनः पदीकृत इहेष्टगुणत्रिजीवा-
भ्यासादनेन विहृता ग्रहलम्बनज्या ।।३२।।

(छायातः पूर्वापररेखा)

जूकाजादिकृतायनार्क भुजजीवान्त्यापमज्येष्टभा
कर्णानां वधतः स्वलम्बकहृतात् त्रिज्याहृता भाभुजा ।
संयुक्तान्तरितस्वदेशपलभाभाग्रान्निजांशोन्मुखा
तच्छङ्कोर्मुखमूलयोर्विलिखिता रेखा हि पूर्वापरा ।।३३।।

(देशान्तरसंस्कारः)

समरेखोदगवाग् या लङ्कारामेश्वरोज्जयिन्यमर्त्याद्रीन्[1] ।
अवगाह्य वर्ततेऽस्याः स्वर्णं देशान्तराख्यकर्म[2] पश्चात् प्राक् ।।३४।।

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1. A_1 न्यमर्तगिरीन्
2. A_1 देशान्तरं हि

धूर्धूरगात् कुपरिधेर्निजलम्बकघ्नात्
त्रिज्याहृतं परिधियोजनमिष्टभूमेः ।
नाड्यादिदेशविवरं निजवेश्मरेखा
मध्यस्थयोजनहतोक्तिरनेन भक्ता ।।३५।।

तन्त्रानीताद् देशसंस्काररहीनात् स्पर्शादेर्भाप्तस्य दृष्टस्य चास्य ।
भेदा सूर्यो दृष्टकालेऽल्पके दिक्[1] पश्चात् तद्देशान्तरं हृच्छ्र्योऽत्र ।।३६।।

(स्वदेशराशिप्रमाणम्)

मेषादिभान्तेष्वयनान्वितेषु कृत्वा चरप्राणकलान्तरे स्वे ।[2]
स्वाद्योनभान्तांशषडंशनाड्यो भमानमानन्दयतोऽमुनाच्छित् ।।३७।।

(दिनरात्रिप्रमाणम्)

कृतायनेऽर्के[3] ऽजतुलादिगेंऽशे[4] स्वर्णं नदीपाप्तचरं द्युमानम् ।
निशा तदूनोक्तिरनर्क्षनाड्यः स्वमाननिघ्नाङ्गहृताः स्वकाः स्युः ।।३८।।

(नाडीकरणम्)

चरासु लिप्तिकाभिदा कृतायनाढ्यभार्कयोः ।
इनोनभांशदिग्वधस्त्विनोदयाद् विनाडिका: ।।३९।।

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1. $B_{2,3}$ कालेऽल्पदिक् (wrong)
2. $A_{1,2,3}$ कृत्वात्म (A_3 कृतात्म) लिप्तासु भिदा चरेषु
3. $A_{1,2,3}$, $B_{2,3}$ कृतायनार्के
4. A_1 तुलादिशेंशे ; $B_{2,3}$ तुलादिगेहे (wrong)

(उदयलग्नानयनम्)

लिप्तासुभेदचरसंस्कृतसायनार्क
भागे त[1]षड्घ्नघटिका: स्वम् अत: पृथक्स्थात् ।
नीत्वाऽऽदिमे विनिमयेन कृताविशेषे
व्यस्तं च कालभ इह व्ययने विलग्नम् ।।४० ।।[2]

(विविधज्या:)

काननं-चलन-वैनतेय सकृदुक्तवाक्-तरळताळसू:
पर्वताळिशुभतेति षट्सु परिधौ गुणोदयबहुश्रुते: ।
इष्टचापकृतिनिघ्नम् अङ्घ्रिकृतिभक्तमाद्यमुपरि त्यजेत्
भूय एवमथ[3] षष्टतो धनकृतं धनुष्यपि गुणाप्तये ।।४१ ।।[4]

षड्वाक्यानि मुनि: फणात्र-खळकेळि-मार्गिचोदी नरो
मुग्धाक्षीतिलमात्रनुत् मननसद्बिम्बोष्टपस्तेष्वध: ।
इष्टेष्वासकृतिघ्नमंघ्रिकृतिसंभक्तं त्यजेत् स्वोपरि
स्वोपर्यन्तकृतं गुणोदयबहुप्रीतौ हृतौ सायक: ।।४२ ।।[5]

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1. A_2, B_1 भागेषु
2. B_4 transfers this verse to the end of the chapter.
3. B_4 एवमपि
4. $B_{1,2,3}$ do not have this verse.
5. $B_{1,2,3}$ do not have this verse.

दोर्ज्येष्टैव क्रमज्या कृतिविरपदं तत् त्रिमौर्व्याश्च कोटि-
स्ताभ्यांत्रिज्याहताभ्यां क्रमश इह बहिर्वृत्ततोऽब्ध्यश्रवेधे ।
स्पृग्ज्या कोटीविभक्ता क्रमगुणविहृता कुस्पृगाख्या च मौर्वी
तद्वद् व्यासार्धवर्गात् पृथगपि भवतः छेदिकुच्छेदिजीवे ।।४३।।[1]

इति शङ्करवर्मनिर्मितायां सद्रत्नमालायां
चतुर्थं प्रकरणम् ।[2]

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1. A_2 has the following lines after this verse :

त्रिज्याघ्ने भुजाकोटी कोटिदोर्भक्ते क्रमात्
स्पृज्ज्याकुस्पृग्ज्ये स्यातां स्पृग्ज्याढ्यं ततस्ततः ।
स्वाप्तकुस्पृग्ज्याप्तं त्र्याद्योज्याप्तं द्वितीयाद्यं
त्यक्तेन्त्ये स्वोर्धात् भूयः शिष्टमिष्टज्या धनुः ।।

2. A_1 इति सद्रत्नमालायां शङ्करवर्ममहाराजविरचितायां तृतीयं प्रकरणम् ।
A_2 इति सद्रत्नमालायां तृतीयं प्रकरणम् ।
$A_3, B_{1,4}$ इति सद्रत्नमालायां चतुर्थं (B_4 तुरीयं) प्रकरणम्
(B_1 adds समाप्तम्)
$B_{2,3}$ इति श्रीशङ्करवर्मनिर्मितायां सद्रत्नमालायां तुरीयं प्रकरणम् ।

अथ पञ्चमं पञ्चबोधप्रकरणम्

(पञ्चबोधात्मकं गणितम्)

छाया-ग्रहण-चक्रार्ध-मौढ्य-शृङ्गोन्नति-श्रिताः ।
पञ्चबोधास्तद्गणितप्रकारस्त्वथ कथ्यते ॥१॥

(सूर्यच्छायागणितम्)

(काललग्नानयनम्)

इष्टः सायनभास्करः स्वचरलिप्ताः स्वन्तराभ्यां कृतः
प्रत्यूषेस्तमये तु षड्भसहितस्ताभ्यामथो संस्कृतः ।
मध्याह्ने कृतलिप्तिकासु विवर[1]श्चक्राङ्घ्रियुक् कालभं
षड्घ्ना स्वोर्ध्वघटीः क्षिपेन्निजलवे तत्कालजं स्यादिदम् ॥२॥

(महाच्छायानयनं तद्द्वारा कालनिर्धारणं च)

कृतायनेष्टार्कचरापमेष्वोर्द्वितीयहीनस्त्रिगुणा द्युजीवा ।
निरक्षसूर्योदयकाललग्नहीनेष्टभूकालविलग्नदोर्ज्या ॥३॥

दिशा चराढ्यान्तरिता द्युमौर्वीनिघ्नाः स्वहारेण हतार्कशङ्कुः ।
तत् त्रिज्ययोर्वर्गभिदा पदं भा स्वोच्चाहता शङ्कुहृता स्वभा स्यात् ॥४॥

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1. B_1 विवरं

छायेष्टशङ्कुकृतियोगपदेन शङ्कु-
त्रिज्यावधाद्धृतहतस्वहराद् द्युमौर्व्या ।
लब्धं चरेण कृतमस्य धनुश्चरेण
व्यस्तं कृतं ह्यतुलहृद् द्युगतैष्य[1] नाड्यः ।।५ ।।

(पलाङ्गुलानयनम्)

त्रिज्याहर्दलभावधाच्छ्रुतिहृताद् भादिग्धनुः सायन-
ब्रध्नापक्रमचापयोगविवरं दिक्साम्यभेदेऽस्य[2] यत् ।
तज्जीवाक्षगुणोऽस्य वर्गरहितत्रिज्याकृतेः स्यात् पदं
लम्बज्याक्षगुणाद् वितस्तिगुणिताल्लम्बाहृताक्षप्रभा ।।६ ।।

(शङ्क्वग्रं अर्काग्रा च)

पलाङ्गुलघ्नार्कनराद् रयाप्तं शङ्क्वग्रमस्तोदयसूत्रतोऽवाक् ।
स्यात्[3] सायनादित्यभुजागुणोऽर्काग्रान्त्यापमघ्नो निजलम्बभक्तः
।।७ ।।

(समशङ्कोः रविस्फुटः)

अर्काग्रालम्बकाभ्यासः क्रान्तित्रिज्यावधोऽथवा ।
समशङ्कुः पलज्याप्तः स्वल्पे सौम्येऽक्षतोऽपमे ।।८ ।।

समशङ्कोः पलज्याघ्नादन्त्यक्रान्तिहृताद् धनुः ।
तत्कालसायनरविर्यद्वा चक्रार्धशोधितम् ।।९ ।।

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1. $B_{2,3}$ द्युगदैष्य (wrong)
2. $B_{2,3}$ भेदस्य (wrong)
3. B_1 स्युः

(छायात: रविस्फुट:)

मध्याह्नार्कनतं खमध्यत उदग् वाऽवाङ् महाभाधनु-
स्तुल्यं यत् क्रमशोऽक्ष[1]चापयुतविश्लिष्टं गुणोऽस्यापमः ।
तत्त्रिज्यावधतः परापमगुणेनाप्तस्य चापं स्वयं
षड्भाढ्यं नु भचक्रतद्दलविशुद्धं वा रविः सायनः ।।१० ।।

इति सूर्यच्छायागणितम् ।[2]

(चन्द्रच्छायागणितम्)

(चन्द्रविक्षेपक:)

इष्टकाले रवीन्दूच्चपातान् दृशा
सायनांशान् नयेत् कालसंज्ञं च भम् ।
आसुरघ्नाद् व्यहीन्दूद्भवाद् दोर्गुणाद्
विस्तरार्धाहतश्चन्द्रविक्षेपकः ।।११ ।।

सत्रिभोच्चोन सूर्यादितोनेन्दुतश्चेन्दुजीवे गृहीत्वैतदभ्यासतः ।
सम्भ्रमाप्तं विधौ स्वर्णमिन्दुज्ययोस्तुल्यभिन्नाशयोरन्त्यचन्द्राप्तये ।।१२ ।।

1. $B_{2,3}$ क्रमशेष

2. Only B_2 has this colophon.

अन्त्यचन्द्रात् त्रिभोनान्नयेद् दोर्गुणं

क्षेपनिघ्नादतो मानसार्थाहृतम् ।

दृक्फलाख्यं धनर्णात्मकं स्यादिदं

दोर्गुणक्षेपदिक् साम्यभेदे क्रमात् ।।१३।।

दृक्फलार्धं विधायान्त्यचन्द्रेऽमुक-

क्रान्तिचापेन्दुविक्षेपयोगान्तरात् ।

ज्ञातदिक्कां चरज्यां द्युजीवां नयेद्

दृक्फलं कृत्स्नमन्त्ये च कार्यं विधौ ।।१४।।

(चन्द्रमहाच्छाया)

स्वासुलिप्तान्तरं चात्र कृत्वा त्यजेत्

काललग्नादमुं तस्य यो दोर्गुणः ।

संस्कृतोऽसौ चरेण द्युजीवाहतः

स्वीयहारोद्धृतः शङ्कुरिन्दोर्भवेत् ।।१५।।

(इष्टचन्द्रच्छाया)

अत्र शङ्कावजादौ शशी दृश्यते

तत्त्रिमौर्व्योस्तु वर्गान्तराद् यत्पदम् ।

इष्टशङ्कून्नतिघ्नादतस्तत्प्रभा-

निघ्नगत्यंशहीनेन्दुशङ्कूद्धृता ।।१६।।

(चन्द्रच्छायात: इष्टकाल:)

इष्ट: शङ्कुर्भवति निजभा यस्य मेया स मौर्व्यां ।
यद्वा श्रीशाङ्गुलमितनर: सार्धषट्पादतुङ्ग: ।
चन्द्रच्छायानुमितसमये कार्यमा संस्कृतेन्दो:
प्राग्वत् कर्म श्रुतिरिह च भाशङ्कुवर्गैक्यमूलम्
।।१७।।

छायावर्गाद् वनहतगतेर्भागनिघ्नात्तदाप्तं
शङ्कुत्रिज्यावधयुतमत: कर्णभक्तोऽब्जशङ्कु: ।
शङ्कोरस्मात् स्वविषयहरेणाहताद् द्युज्ययाप्तं
जूकाजादिस्वचरगुणयुक्तोनितं चापितं यत् ।।१८।।

प्राक्तद्व्यस्तं कृतचरमथो षड्भशुद्धं च पश्चात्-
न्यायाद्वाभ्यादपि च कृतचरे संस्कृतेन्दौ क्रमात् स्वम् ।
तत्सूर्यास्तोदयगदितकालर्क्षभेदद्वयांशा:
षट्भिर्भक्ता निशि च गतगन्तव्यनाड्योऽविशिष्टा: ।।१९।।

क्रमेणैवं चात्र क्रमचरकृत: संस्कृतविधु:
स्वयं सत्र्यर्क्षो वा भगणदलयुग् व्यस्तचरक: ।
सम: कालर्क्षेणोदयगगनमध्यास्तगमने
गतिघ्नाद् भेदांशादतुलहृतमिन्दौ स्वमधिके ।।२०।।

।। इति छायाक्रिया ।।[1]

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1. $B_{2,3}$ इति चन्द्रच्छायागणितम्

(ग्रहणगणितम्)

(ग्रहणे सामान्यक्रिया)

पातोनपर्वान्तरवेर्भुजेऽथ गत्यन्तराल्पे ग्रहणं गणेयम् ।
रवेरमान्तेऽहनि पूर्णिमान्ते तत्प्राह्णसायाह्ननिशासु चेन्दोः[1] ।।२१ ।।

नीत्वा विलिप्तान्तमिमौ सभुक्ती पातं च दृश्येष्वयनं च कुर्यात् ।
राकान्तसूर्यस्त्विह षड्भयुक्तः पर्वान्तगौ तौ तु मिथः समानौ
।।२२ ।।

(चन्द्रसूर्यग्रहणयोर्भेदः)

नीत्वा त्रिभोनविधुहारचरासुलिप्ता-
भेदानदश्चरमसंस्कृतसत्रिभेन्दोः ।
कालोनिताच्चरकृतो गुण आत्महार-
पादाढ्यहारविहृतः खलु लम्बनं तत् ।। २३ ।।

संस्कृत्य पर्वणि दिशात्र तथाऽविशिष्टे[2]
पर्वामुना कृतमिनग्रहमध्यकालः ।
मार्ताण्डबिम्बनतलम्बननैरपेक्ष्यात्
राकान्त एव हि विधुग्रहणस्य मध्यः ।।२४ ।।

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1. B_2 निशोर्थ इन्दोः ; B_3 निशोत्थ इन्दोः
2. B_4 तथात्र कृताविशिष्टे

(बिम्बलिप्तानयनम्)

लिप्तात्मिके[1] नवकनाट्यहते स्वभुक्ती
बिम्बौ रवेश्च शशिनश्च कुमारभक्ते ।
दैवघ्नसूर्यगतिहीनशुकघ्नचन्द्र-
भुक्तेः कला त्विह तमो लकुटेन भक्ता ।। २५ ।।

(रविग्रहणे नतिलिप्तानयनम्)

संहारकघ्नपलभार्धमवाक् त्रिभोन[2]
पित्र्यर्क्षकालभगुणेन कृतादमुष्मात् ।
स्वव्यस्त[3] लम्बकनिशावधतः कुजघ्न-
गत्यन्तरांशकहृतेन हृता नतिः स्यात् ।। २६ ।।

क्षेपश्चतुर्घ्नविफणीन्दुगुणाद्यमाप्तो
नत्या कृतोऽयमरुणग्रहणे स्फुटः स्यात् ।
क्षेपेऽधिके युतिदलादिह नोपरागः
[4]पूर्णोन्तरार्धगलिते वलयाकृतिर्वा ।।२७।।

1. B_1 लिप्तात्मके
2. Emended for दिगाक्षं, see notes on V. 26.
3. $B_{2,3}$ सव्यस्त (wrong)
4. $B_{2,3}$ पूर्णे

(इष्टग्रासः)

क्षिप्तीष्टकालकृतलम्बनपर्वभेद-
गत्यन्तरांशवधवर्गयुतेः पदोनम् ।
ग्रासोन्वयार्धमरुणस्य नतेर्दिशीन्दो-
र्व्यस्तं वदेदिह न चाष्टम षोडशौ तौ ॥२८॥

(स्थित्यर्धः)

क्षेपान्वयार्धकृतिभेदपदात्तु भोग
भेदाहृतं स्थितिदलं त्वथ[1] मध्यकालः।
तद्धीनयुक् तदपि लम्बमपीह नीत्वा
कृत्वाथ पर्वणि तथैव कृताविशेषौ ॥२९॥

तौ स्पर्शमोक्षसमयौ च तथान्तरार्धात्
क्षेपाद् विमर्ददलजानयनाविशेषे ।[2]
कालौ निमीलनतदन्यभवौ क्रमेण
तौ नेमिपूर्तिलवजावधिकेऽर्कबिम्बे ॥३०॥

(वलनम्)

पर्वान्तचन्द्रचरसंस्कृतषड्घ्ननाडी
रूपांशयुक् त्रिभगुणोऽक्षगुणोन्तिमाप्तः ।
तत्कार्मुकेन कृतचापितसत्रिराशी
ग्राह्यापमास्य गुणतो वलनं गुणाप्तम् ॥३१॥

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1. A_3 त्विह
2. $B_{2,3}$ विशेषौ (wrong)

(ग्रहणलेखनम्)

दिक्सूत्रनीतवलनाद् वरुणेसरान्तं[1]
क्षेपान्तरे स्वदिशि च स्मृतिमेयरेखे ।
ग्रासोनयोगदलदूरगमन्दगेन्दू
लेख्यौ तयो: स्वदलतो गलदिन्दुरैन्द्र्याम् ।।३२।।

इति ग्रहणक्रिया ।[2]

(व्यतीपातगणितम्)

द्विघ्नायन कृतार्कोनगोलान्तसदृशे विधौ ।
प्रायश्चक्रार्धदोषोऽत्र सायनार्केन्द्वहीन् नयेत् ।।३३।।

चक्रे स्पष्टक्रान्तिगत्या रवीन्द्वोर्बिम्बौ यावत् प्रातिकूल्यं प्रयात: ।
तावत्कालं स्याद् व्यतीपातजन्यो दोष: प्रायेणोपरागेण तुल्य: ।।३४।।

मूलं तु यावदरुणाश्रितभादथाप्या–
ल्लाटो हि तावति मनूडुनि वैधृतोऽस्मात् ।
तुल्यं दृशान्यदखिलं स्फुटभोगनिघ्न:
क्षेपो विधो: परहिते मृदुभुक्तिभक्त: ।।३५।।

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1. A_3 वरूणे स- gap -न्तं

2. $B_{1,2,3,4}$ ग्रहणगणितम्

विक्षेपापमचापयोगविवरं चान्द्रं समानं यदा

सौरक्रान्तिशरासनेन परमासन्नं तु वा युक्तितः ।

सूर्याचन्द्रमसोर्विरुद्धपदयोर्दोषस्य मध्यस्तदा

भेदेऽल्पे निजबिम्बयोगदलतोऽस्यादिस्ततोऽन्तः समे ।।३६ ।।

व्यह्यब्जोत्थान्यायनक्षेपखण्डे चन्द्रक्रान्तेः खण्डतश्चाधिकेऽपि ।

शोध्यक्षेपादल्पकेऽपक्रमे वापीन्दोः पादव्यत्ययः कल्पनीयः ।।३७ ।।

नैवालक्ष्मक्रान्तिसाम्यादिकेऽसौ

दोषः किन्तु क्रान्तिसाम्ये सचिह्ने ।

सम्पर्कार्धादल्पतायां विलक्ष्म-

क्रान्त्योर्भेदस्यापि दोषोऽस्ति तत्र ।।३८ ।।

साम्यं भुजाश्रितविधोरपमेऽधिकेऽधो

भाव्यल्पके विनिमयादिह कोटिगेन्दोः ।

क्षेपापमासमसमायनखण्डभेदै-

क्याप्तापमान्तरहतैकधनुः कलेन्दोः ।।३९ ।।

भास्वीय[1] भोगगुणिता शशिभुक्तिभक्ता

सूर्यस्य सा नखहृता फणिवामलिप्ता ।

एताभिरूर्ध्वमध आनयनं च तेषा-

माद्यन्तमध्यसमयेषु मुहुः क्रियैवम् ।।४० ।।

।। इति व्यतीपातगणितविधिः ।।

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1. $A_{1,3}$, $B_{2,3,4}$ सास्वीय ; B_1 स्वास्वीय (wrong)

(मौढ्यगणितम्)

श्रेय:-सत्य[1]-गया-पयो-धन-शका मूढांशकाश्चन्द्रतो
निःसारान्ध-निरेक-नीति-निरया-त्रेयाः पराः क्षिप्तयः ।
राहुर्नायकनेत्रनक्ररुगिनश्रीर्नाकुला भागभैः
पातास्तद्गुणपातहीनगुणतस्त्रिज्याहृतः क्षेपकः ।।४१ ।।

शीघ्रोच्चाद् बुधशुक्रयोरिनजगुर्वङ्गारकाणां[2] दृशि
स्वोपान्त्यस्फुटतः स्वपातवियुतिः शीतत्विषः[3] स्वस्फुटात् ।
विक्षेपस्तु हतश्चलोच्चरहितः स्वोद्धृतचन्द्रज्यया
भक्तस्तादृगुपान्तिमस्फुटभवश्चन्द्रज्यया स्यात् स्फुटः ।।४२ ।।

मध्यान्निजात्तु कृतमन्दफलाद् विशोध्यः
पातः पुनः परहिते मृगकर्कटादौ ।
क्षेपोऽन्त्यकेन्द्रजनिखण्डगुणोनयुक्त
मारारिवर्धित इहास्तु मुरारिभक्तः ।।४३ ।।

प्रातः कालभसायनेष्टविहगौ कार्याविहाल्पे रवौ
खेटादस्तमये ततः खचरतः स्वक्रान्तिचापं नयेत् ।
तत्क्षिप्त्यो[4]र्हरिदैक्यभेदवशतो योगान्तरोत्थं चरं
स्वार्थाद्यं[5] त्विह तं निजाक्षवधतो राद्धान्तभक्तं निजम् ।।४४ ।।

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1. A_2 सेव्य
2. A_2 जजीवाङ्गारकाणां
3. A_3 श्वेतत्विषः
4. $B_{2,3}$ तद्भुक्तयोः
5. A_2 प्रीताढ्यं (wrong)

खेटे स्वर्णं त्रिभोनस्वभवशशिगुणक्षेपघाताद् वसन्ते-
नाप्ता लिप्ता द्वयाशासमविषमतयाथ स्वलिप्तासु भेदम् ।
क्रान्त्युत्थं चात्र कुर्याच्चरमथ विपरीतं तदस्ते भषट्कं
[1]एवं कालाख्यलग्नान्तरलवगलिते मूढभागे स दृश्य: ।।४५।।

।। इति ग्रहास्तमयक्रिया ।।

(शृङ्गोन्नति:)

(शृङ्गोन्नतिगणितम्)

शृङ्गोन्नत्या[2] च दृङ्मौढ्ये निखिलं कर्म वैधवम् ।
द्वितीयेन्दुं विधायैव विक्षेपानयनं विना ।। ४६ ।।

तिथेर्भुजा कोटिगुणाल्पताडित:
क्षेपोऽधिकाप्त: स्वदिगन्यथा युजि ।
विसंस्कृतेन्दोस्त्रिभयुक्तकालभा-
नीतो गुण:[3] स्वाक्षहतोऽन्तिमाहृत:[4] ।।४७।।

त्रिभ[5]सहितसुधांशोर्ज्यापमोऽप्यत्र नेय:
समविषमहरित्वाच्चापितानाममीषाम् ।
युतिविरहजजीवा चन्द्रबिम्बार्धनिघ्ना
त्रिभगुणविहृतेयं चन्द्रशृङ्गोन्नति: स्यात् ।।४८।।

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1. Emended for एवं चैतत्कालग्नान्तर . . . दृश्य: ।
2. $A_{1,3}$ शृङ्गोन्नतौ
3. $B_{2,3}$ गुण
4. $B_{2,3}$ °न्तिमाप्त:
5. B_1 सहिताविधोश्चज्या

व्यर्कचन्द्रनक्रकर्कटादिकोटिमौर्विको
नाड्यविस्तरार्धचन्द्रमण्डलार्धघातकः[1] ।
त्रिज्यया हृतं सितं स्वमण्डलार्धवर्गतो
द्व्यन्तराप्तमन्तराढ्यमत्र सूत्रमर्धितम् ॥४९॥

दिग्व्यासात् स्वदिशीन्दुपूर्वपरिधिस्पृक् शृङ्गतुङ्गाग्रक[2] -
स्पृग्व्यासे परिधेः सितं मघवपाश्यग्रं च पक्षक्रमात् ।
अन्तर्न्यस्य तदग्रपार्श्वपरिधिस्पृग्वृत्तखण्डं लिखेत्
सूत्रेणान्तसितासितेन शशिबिम्बार्धाधिकाल्पे सिते ॥५०॥

(इति शृङ्गोन्नतिः)

श्रीलोकाम्बाकटाक्षात् कथितमिह मया पञ्चबोधप्रकारं
ये सम्यक् शीलयन्ति प्रथितगणितसिद्धान्तसर्वस्वसारम् ।
लीलाभेदा यदीया गणितविषयरूपाः स दिक्कालरूपः
श्रीकृष्णः कृष्णमेषाह्वयनिलयलसन्नेषु पुष्णातु लक्ष्मीम्[3] ॥५१॥

इति श्रीशङ्करवर्मविरचितायां सद्रत्नमालायाम्
पञ्चमं पञ्चबोधप्रकरणम् ॥[4]

1. B_2 घातितः ; B_3 घाततः
2. A_2 ग्रग ; B_1 Hapl. om. स्पृक् (....) वृत्तखण्डं two lines below.
3. A_2 omits the verse.
4. A_1 चतुर्थ प्रकरणम् ; A_2 तुरीयं प्रकरणम् ; A_3 B_1 पञ्चमप्रकरणम् ; B_2 पञ्चप्रकरणम् ।

अथ षष्ठं गणितपरिष्करणप्रकरणम्

(आर्यभटस्य भूदिनपर्ययादीनि)

आचार्यार्यभटप्रणीतकरणं प्राय: स्फुटं तत् कलौ
गोत्रोत्तुङ्गमिताब्दके व्यभिचरत् ब्राह्मादिसिद्धान्तके ।
भूघस्रोऽत्र चतुर्युगस्य नृनमत्सत्केळिसार्थाशियो
नानाज्ञानफलौघ एष भगणोऽर्कस्य ज्ञभृग्वोरपि ।।१ ।।

चन्द्रार्कीड्यासृजां षड्बलगुण[1] सुसृणि:[2] श्वेतमत्तेभपत्नी
विप्रेन्द्रो वृत्तिलग्नो ज्वरदिषुधिखरश्चित्ररेखाम्बरोऽहे: ।
इन्दूच्चस्य भ्रमो धिक् कुरु हृदघमिति ज्ञाच्छशीघ्रोच्चयोर्ज्ञ
श्रीनाथो बुद्धिसेव्यो हृदिगुरुरिनसू: सौर एवेतरेषाम् ।।२ ।।

भूघस्रै: कलिघस्रपर्ययवधाल्लब्धो गत: पर्यय:
शिष्टा द्रव्यनगोक्तिभि: क्रमहताद् राश्यादिको मध्यम: ।

(परहितगणितम्)

दृग्वैषम्यवशाद् महास्थलमिते कल्यब्दके निश्चित:
संस्कारो विबुधैर्यत: परहितत्वं तेषु वीनेष्वयम् ।।३ ।।

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1. A_3, $B_{1,2,3}$ गण

2. $A_{2,3}$ सुसृणी ; B_2 सुसृणी:

(शकाब्दसंस्कार:)

ज्ञोच्चार्क्यारा निरूढीनखशुभनिहतात् गोत्रतुङ्गोनकल्य–
ब्दादाढ्या मागरै: सावन गणकहतात् सेड्य[1] ऊनोच्छतुङ्ग: ।
धीशान्ति: श्लोकनिघ्नान्मदविलयफलैराप्तलिप्ताभिरूना –
श्चन्द्र: सत्र्यर्क्षतुङ्गो भगणदलविशुद्धोरगश्च क्रमेण ।।४ ।।

अयनयुगिनदिक्कं[2] साक्षदेशेष्टकाले
चरमखिलमहर्मानेऽथ तद्व्यक्षकाले ।
अहनि निशि घटी मायान्तरघ्नं मयाप्तं
चरमिह दिनमध्याद् व्यस्तदिक् चानिशीथम् ।।५ ।।

(इष्टकाले शकाब्दसंस्कार:)

देशान्तरासुरविदो: फलसायनार्क[3] –
लिप्तासुभेदचरयोगभिदातुलांश: ।
व्यस्तं तु पर्वघटिकासु यदीष्टकाले
स्वर्णं दिशा गतिहतो यदि मध्यमादौ ।।६ ।।

एकधैवेष्टकाले स्याद् बहुधैवंकृतो ग्रह: ।
वर्ज्यं[4] परहितोक्ताहर्मानस्यासुकलान्तरम् ।।७ ।।

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1. $A_{2,3}$ दीढ्यु ; B_1 साध्य
2. A_3 युक्तं
3. A_2, $B_{1,2,3,4}$ सायनेन
4. B_1 ग्राह्यं

(ग्रहमन्दोच्चानि)

भास्करस्येह[1] मन्दोच्चं दैत्यारिर्भागराशिभिः ।
भौमाज्जरागो[2] नानार्थावनन्ता[3] नङ्गषड्रसाः[4] ॥८॥

(ग्रहाणां मन्दशीघ्रपरिधयः)

विद्या हृद्या सूनुर्मानि स्थानी जह्नुर्भानुर्ब्रध्नः ।
धेनुर्गोप्या मन्दे[5] केन्द्रे दोराद्यन्ते वृत्ते भौमात् ॥९॥

शैघ्रे केन्द्रे दोराद्यन्ते वृत्ते तद्वल्लक्ष्मीकृष्णौ
योगी धीरस्तार्क्ष्यो मान्यो धर्मः सूक्ष्मो धेनुर्दीना[6] ॥१०॥

(ग्रहाणां स्फुटपरिध्यानयनोपायः)

दोर्ज्यावृत्तान्तराभ्यासत्रिज्यांशाढ्योनितं क्रमात् ।
आद्यवृत्तं परादल्पाधिकं[7] स्यात् परिधिः स्फुटः ॥११॥

स्फुटवृत्तं सदा भानोर्गानं सूनं विधोरपि ।
वृत्तघ्नदोर्ज्या नन्दांशचापोऽर्कान्मन्दमौर्विकाः ॥१२॥

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1. $B_{2,3}$ भास्करस्यैव
2. $B_{1,2,4}$ जरागौ
3. A_3 ज्ञानाक्षोनन्त
4. B_2 षड्रसम् ; B_4 षड्रशः
5. $B_{1,2,3,4}$ मान्दे
6. B_2 दीनः
7. A_1 धिकः

दोः कोटिज्ये स्फुटपरिधिनिघ्ने नदाप्ते फले ते
कर्क्येणादावृणधनमिदं कोटिजन्यं त्रिमौर्व्याम् ।
तद्वर्गाढ्यादिमफलकृतेः स्यात् पदं शीघ्रकर्ण-
स्त्रिज्याघ्नाद्याच्छ्रुतिहतधनुः कर्किनक्रादिजीवाः ।।१३ ।।

मन्दगुणोत्पन्नगुणान्नन्दगुणाद्यद्विहृतम् ।
तद्गुणदोमौर्विकया वृत्तमिदं तत्रभवम् ।।१४ ।।

कर्क्येणादेमौर्विकायास्तद्धोर्युतमुचा ज्यया ।
नन्दघ्नान्मौर्विकाज्यातः स्वशीघ्रपरिधिर्हृतः ।।१५ ।।

अन्त्यदोःफलजं वृत्तं दोष्णः परिधिरन्तिमः ।
एकर्क्षदोःफलोद्भूतं द्विघ्नमन्त्योनमादिमः ।।१६ ।।

(ग्रहस्फुटोपायः)

मन्दोच्चोनितमध्यजार्धितमृदुज्यासंस्कृते मध्यमे
शीघ्रोच्चोननिजोत्थशीघ्रजदलं कृत्वा मृदूच्चं त्यजेत् ।
मध्ये कार्यममुष्य मन्दजफलं कृत्स्नं पृथक्स्थाच्चलो -
च्चोनाच्छीघ्रफलं च तद्वदिनजेड्यारस्फुटावाप्तये ।।१७ ।।

कर्माण्येवमनादीनि बुधशुक्रस्फुटाप्तये ।
मध्यौ रवीन्द्वोः स्वोच्चोनस्वदोः फलकृतौ[1] स्फुटे[2] ।।१८ ।।

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1. $B_{2,3}$ कृते
2. A_3, B_2 स्फुटौ

(ग्रहाणां स्फुटगतिः)

उन्नतपुरघ्नभगणावनिदिनांशो
मध्यगतिरत्र तरणेः स्वगतिसिद्ध्यै ।
कर्किमकरादिनिजकेन्द्रगुणकोट्या :
स्वर्णमणिमाद्यहृतमंशु[1] हृतमिन्दोः ।।१९ ।।

कुलीरमकरादितो मृदुगुणान्तरघ्नात् स्वतः
शरासशकलोद्धृतं मृदुगतौ धनर्णं पुनः ।
स्वशीघ्रगतिभेदशीघ्रगुणखण्डघातात् त्वृणं
धनं स्फुटगतिर्भवेत् वरगुणेऽल्पके व्यत्ययात् ।।२० ।।

(स्फुटसङ्क्रान्तौ आदित्यमध्यमः)

उच्चोनर्क्षान्तभानुस्फुटमकरकुलीरादिकोटीफलोना–
ढ्यान्त्यावर्गाढ्यतद्दोः फलकृतिपद[2] मत्रोदितो व्यस्तकर्णः ।
व्यासार्धाद् दोःफलघ्नाच्छ्रुतिहृतफलचापं क्रमाद् भान्त[3] भानौ
जूकाजादावृणं स्वं भवति पुनरसौ संक्रमोत्थार्कमध्यः ।।२१ ।।

(राशिषु नक्षत्रेषु च आदित्यगतिः)

स्पष्ट[4] संक्रान्तिकालार्कमध्योंऽशितो दोः फलेनार्कवर्षान्तजेनान्वितः ।
भूदिनघ्नो हृतः सौरघस्रैर्भवेन्मासवाक्यं तथार्कोडुचारोद्भवम् ।।२२ ।।

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1. B_3 माद्यममुमंशु
2. $B_{2,3}$ फल for पद
3. $A_{2,3}$, $B_{1,2,3}$ भान्त्य
4. A_3 स्फुट

(संवत्सरवाक्यानां राशिनक्षत्रवाक्यानां च गणनम्)

गुर्वक्षराद्यः शुकयोगिमान्यमार्ताण्ड एकादिहतोऽब्दवाक्यम् ।
मासोडुचाराब्दसदाप्ति शिष्टं संक्रान्तिवाक्यं निजमर्कवारात् ।।२३ ।।

(योग्यादिवाक्यगणनयुक्तिः)

दिनाष्टकतदाद्यन्तस्पष्टार्कान्तरयोर्भिदा ।
मुहुर्मेषादि योग्यादि स्वर्णमल्पेऽधिके दिने ।।२४ ।।

(रवेः सङ्क्रमस्फुटः)

भास्वद्दोःफलचरनीवृदन्तरासून्
नीत्वैषां युतिविवरं दशाहतं यत् ।
वाक्ये स्वे ध्रुवयुजि संक्रमस्फुटाप्त्यै
संस्कार्यं विनिमयतो विनाडिकाधः ।।२५ ।।

(चान्द्रसौरमासदिवसादिः)

चान्द्रा मासारवीन्द्वोर्भगणविवरमर्कभ्रमो द्वादशघ्नः
सौराश्चा[1] थाधिमासास्तदुभयविवरं तेऽम्बुनिघ्ना द्युरूपाः ।
ज्ञेयश्चान्द्रो[2] दिनौघः[3] क्षितिदिनरहितोऽत्रावमाख्यस्तिथीनां[4]
कल्पाश्चोर्वीदिनालीयुगरविभगणैक्यं च नाक्षत्रघस्राः ।।२६ ।।

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1. B_2 Hapl. om श्च (थाधि... श्चा) न्द्रा, one line below ; B_3 contains the omitted letters.
2. B_1 एवं चान्द्रो ; B_2 चान्द्रा
3. B_2 दिनौघा
4. A_2, B_2 माख्या तिथीनां

(ग्रहकक्ष्यानयनविधिः)

चन्द्रभ्रमाननचकोरहतिः स्वकक्ष्या
सूर्यादिपर्ययहृतास्तु ततः स्वकक्ष्याः ।
क्षोणीदिनैस्तु दिनयोजनभुक्तिनाप्ता
कक्ष्या रवेर्नुतिहताश्विमुखर्क्षकक्ष्याः ॥२७॥

(ग्रहाणां बिम्बव्यासानयनयुक्तिः)

योजनरूपो बिम्बव्यासो वह्निमयार्कस्योद्यद्भावः ।
शकलोऽब्रूव प्रालेयांशोर्मृण्मयभूमेरात्मा नित्यः ॥२८॥

विष्कम्भदूराहति[1] दूरवर्गयोगस्य मूलं खलु दृश्यसीमा ।
व्यासार्धदूरैक्यकृतिर्विहीना व्यासार्धकृत्या पदिताऽपि गोले ॥२९॥

दृष्ट्युन्नतिकृतिरहितां समभूपृष्ठस्य दृश्यसीमकृतिम् ।
दृष्ट्युन्नत्या विभजेल्लब्धं भूगोलमध्यविष्कम्भः ॥३०॥

(भूपरिध्यानयनम्)

स्वमध्ययोजनाभ्यस्ता याम्यसौम्येष्टदेशयोः।
स्वाक्षचापान्तरांशाप्ताश्चक्रांशाः परिधिर्भुवः ॥३१॥

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1. $B_{2,3}$ मूलाहति (wrong)

(सूर्यादीनां कक्ष्यानयनम्)

प्राग्वदिष्टार्कोत्पन्नव्यस्त[1]कर्णाप्ता[2]न्तिम-
ज्याकृतिर्मृदुश्रुतिः सार्ककक्ष्ययाभ्यस्ता ।
चक्रलिप्ताप्ता भवेत्[3] स्फुटयोजनश्रुतिः
सेयमुष्णांशोः कक्ष्याव्यासार्धं[4]तात्कालिकम्[5] ।।३२ ।।

ज्ञोच्चादे[6]र्मागराद्याः पृथगवनिदिनघ्ना हराः पर्ययघ्नाः
ज्ञानीन्द्रघ्नैर्निरूढ्यादिभिरपि युतमुक्ता गुणाः संयुतोऽहः ।
कल्यादावत्र राशित्रययुतविधुतुंगेऽप्यहौ षड्भशुद्धे
व्यस्तं कार्या निरुढ्यादिहतगिरितलान्मागराद्याप्तलिप्ताः ।।३३ ।।

सम्प्रोक्ता गुणका इह क्षितिदिनक्षुण्णा स्वहारोद्धृता
वेद्याः संस्कृतपर्ययाश्च गुणहारास्तेऽपवर्त्या अपि[7] ।
इष्टाहर्गण एव खण्ड इह खेटानां स्वमध्यं ध्रुवां
ग्लौवाक्यानि पृथक्कृतस्फुट शशाङ्का देवरान्तैर्दिनैः[8] ।।३४ ।।

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1. B_1 gap indicated for स्त
2. $B_{2,3}$ omit प्ता
3. A_3 omits भवेत्
4. B_3 व्यासार्धः
5. B_3 adds इतः परं न व्याख्यातम् ।
6. A_2 ज्ञोच्चादे
7. $A_{2,3}$ मध्या
8. $A_{2,3}$ देवरातैर्दिनैः

इष्टो गुण: स्वभगणेन हृतं गुणघ्न[1]
क्षोणीदिनादपि हर: कुदिनघ्नहारात् ।
तत्रोनशेषहृतचक्रकलाविभक्तं
स्वर्णात्मको भवति चास्य हरो द्वितीय: ।।३५ ।।

मिथो हृतगुणच्छिदो: फलमधोऽध एकत्रत-
न्मुखोनमपरत्र चैकमुभयस्य चोर्ध्वं न्यसेत् ।
तृतीयफलत: स्वमूर्ध्वगुणितं[2] तदूर्ध्वान्वितं[3]
तयो: प्रथमगोर्ध्वगं त्यजतु ते च हारा गुणा: ।।३६ ।।

द्वौ मिथो हरणोऽधिकेन हृतौ दृढावपवर्तितौ
हार संगुणितौ मिथ: सहरौ गुणौ समहारकौ ।
भूदिनं शशितुङ्गयोर्भगणान्तरं च मिथो हरेत्
तै: फलैर्विहितास्तथा[4] गुणहारका: शशिकेन्द्रजा: ।।३७ ।।

केन्द्रहृद्गुणकलिकृतोदय विलिप्तिकादिसुकळाम्बुयुक्
चन्द्रकेन्द्रत[5] इनातपत्रहृतमूर्ध्वहारगुणितं च यत् ।
ओजयुग्ममयकेन्द्रहृद्धृत इहाधिकोन यु[6]गहर्गणो
वाक्यखण्ड उडुपस्फुटं निजकृतं ध्रुवोऽस्य च हृदां तथा ।।३८ ।।

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1. A_3 गुणघ्नं-
2. A_3 गणितं
3. A_3 तमूर्ध्वान्वितं
4. B_4 comments after the gap.
5. B_1 omits केन्द्र and reads चन्द्रत
6. $A_{1,3}$, B_4 मु for यु

हार्यः स्याद् ध्रुवसंस्क्रिया हरकृतौ केन्द्रोत्थ इष्टो गुण-
स्तत्तद्धारमिथो हृतौ समधिकाःपङ्क्त्या[1] मृणस्वात्मकाः ।
सौरद्यूर्ध्वगहार[2] लब्धफलगुण्याः खण्डचन्द्रोच्चयो-
र्भेदघ्नः स हरो नतत्परहतश्चन्द्रे धनात्माधिके[3] ।।३९ ।।

कल्यब्दनिघ्नोऽधिमासः सूर्यपर्ययसंहृतः ।
भूदिनघ्नोऽथ कल्यादिचन्द्रध्रुवलवाहृतात्[4] ।।४० ।।

भूदिनादतुलाप्तोनोऽधिमासाप्तस्वखण्डकः ।
अन्योन्याप्ताधिमासोर्वीदिनोत्थास्तद्धराहृताः[5] ।।४१ ।।

मध्ययोः स्फुटयोर्वा[6] न्तावमयोर्यदि मध्यगौ ।
तथाविधार्कसंक्रान्त्योरधिमासाश्चतुर्विधाः ।।४२ ।।

('जनसभा' 4708 मितकल्यब्दान्ते दृक्समकरणादिः)

प्रत्यक्षतः परहिते गणितेऽपि भेदं
दृष्ट्वा[7] कलौ जनसभामितवत्सरान्ते ।
राद्धान्तिते तु गणकैर्गणितागमे यद्
दृक्साम्यगामि करणादि तदत्र वक्ष्ये ।।४३ ।।

1. A_1 gap for ङ्क्त्या
2. $A_{1,3}$ र्ध्वहरार (wrong)
3. B_1 धनात्मिके
4. B_1 हतात्
5. B_1 हताः
6. A_2 gap for र्वा
7. B_1 कृत्वा

ज्ञाननिघ्नफलविन्नराङ्गगुणसत्सुमं रुरुपदार्जवं[1]
लक्षदातृधरराडनेकसुगलोत्सुकः क्षितिपभूतलम् ।
श्रीसखीकुरनसास्फुटाक्षतघटो दधत्खरगरः क्रमात्
भास्करादिभगणाश्च भूदिनमनीशसायुधसुसंशयः ।।४४।।

आज्ञातत्परतो हृतं हरिहयासन्नास्य[2] भीमार्भकैः
मालाशोभि जलार्थि[3] रत्ननृपदैत्यारीड्यनारीस्तनैः ।
कल्यादिध्रुवमंशकादिकुजसौम्यार्कीष्वृणं स्वं क्रमा-
च्चन्द्रे सत्रिभतुङ्गचक्रदलशुद्धाह्योश्च जीवाच्छयोः ।।४५।।

भात्युदयाद्रिर्भानुमृदूच्चं भूतनयादेर्बुद्बुदनाभः ।
संघटनार्थं मुनिरुद्रांशो निभर्यराष्ट्रे धेनुरनन्दत् ।।४६।।

व्यासाः कुजादेर्यजमाननासा पुटीधमत्[4] कीटसुनाडिकान्धजाः ।
गुणाद् दशघ्नात् त्रिजगन्नदीन[5] विस्तारमायाकळभाप्तमृग् युताः ।।४७।।

व्यासास्तु शीघ्रपठिता[6] उपमेयराजमन्त्री द्युनिर्मित[7] तमोलयनाळदेहाः ।
आढ्या[8] विदेशगुळिका ध्वनिकृच्चिकित्सा योगैर्दशघ्नगुणतः क्रमशश्च लब्धैः
।।४८।।

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1. B_1 पदार्णवं
2. B_1 सन्नाढ्य
3. B_1 जनार्थ
4. A_3 पुधीनमत्
5. A_3 नदीर्न
6. A_3 शीघ्रवनिता
7. A_3 दुर्वित
8. $B_{1,4}$ ऊना for आढ्य

मन्दशीघ्रभुजाकोटिगुणाः शतसहस्रशः[1] ।
स्वव्यासाभ्यां फलान्यान्यत् सर्वं परहितोक्तवत् ।।४९ ।।

[2]कर्क्येणादिकमन्ददोःफलजकोट्या स्वीयकोटीफल-
स्वर्णिन्या विहृतार्ध[3]विस्तृतिकृतिर्मन्दश्रुतिस्तद्गुणात् ।
कक्ष्याव्यासदलाहताच्चचलकर्णात् संहृतं ह्यन्तिम-
ज्याकृत्या स्फुटयोजनश्रुतिरिळाजादेर्विधोः सूर्यवत् ।।५० ।।

हृतानि शशिबिम्बतस्तट[4] रट[5]न्नटीहृद्भटैः
कुजादिनिजयोजनान्युडुपकक्ष्यकेऽक्षीयाणि च ।
स मुक्तनिजबिम्बयोजनहतत्रिमौर्व्याः पुनः
स्फुटाख्यनिजयोजनश्रुतिहृतं त्विनादेः कलाः ।।५१ ।।

मार्ताण्डस्फुटयोजनश्रुतिहताद् भूव्यासतः सूर्यभू -
विस्तारान्तरसंहता खलु तमः सूची[6] कुगोलार्धभूः ।
तच्चन्द्रस्फुटयोजनश्रुतिभिदा भूव्यासघातात् तया
भक्तं[7] व्यासदलघ्नमुक्तशशिकर्णाप्तं तमोलिप्तिकाः ।।५२ ।।

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1. $B_{1,4}$ शतगुणाहृताः
2. The verse as given here does not explain *vargaikyamūla.* Hence it may be corrected as:
   खेटस्य स्फुटकोटिमन्दफलयोर्वर्गैक्यमूलं ततो तेन स्याद् विहृतार्ध . . . .।
   For more details, see Notes under this verse.
3. A_2 विहितार्ध
4. B_1 स्फुट
5. $A_{1,3}$ रद
6. $A_{2,3}$ सूर्यात् for सूची ; A_1 omits ची; B_1 omits कु following.
7. A_3 भग्नं

द्विनिघ्नपातारुणपर्ययान्वयं युगेन्दुमासांश्च परस्परं हरेत् ।
लब्धोत्थहाराः कुदिना[1] हताश्च तै र्मासैर्विभक्ता ग्रहणोक्तहारकाः ।।५३ ।।

मध्यार्केन्द्वोर्दिशि विफणिनोर्मध्यपर्वान्तभाजो-
भागाल्लूनाञलिविनिहते[2] चक्रभागैर्विभक्ते[3] ।
[4]तर्कार्थघ्ने लुनदगहृते[5] शिष्टभूघस्रघाताद्
ग्लौमासाप्तोनित[6]दिनगणैश्चोपरागोक्तखण्डः ।।५४ ।।

(इष्टकाले गणितपरिष्करणोपायः)

तन्त्रोद्भूतपरिक्षितग्रहभिदा लिप्ताहतक्ष्मादिना-
लब्धं व्यन्तरवासरोन्नतपुराभ्यासेन तत्पर्यये ।
स्वर्णं तत्रभवेऽल्पके समधिके चार्कस्य नायं विधि-
स्तत्काले द्युगणः[7] परीक्षणभवो मध्यश्च खण्डध्रुवौ ।।५५ ।।

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1. A_1 कुलिना
2. $B_{1,4}$ विनिहितात्
3. A_1 विभक्तो
4. B_4, a few letters broken off here.
5. A_3 पुनरगहृते
6. B_4 breaks off here, the last leaf having been lost.
7. A_3 भगणः

(ग्रन्थकरणकालः)

प्राक् सृष्टेः प्रलयात् परं च युगषट्कार्धे च कल्पे विधेः
प्रत्येकं मनवश्चतुर्दश च सप्तत्या युगैः सैकया ।
अष्टाविंशयुगेऽत्र सप्तममनोर्वैवस्वतस्यान्तरे
तुर्याङ्घ्रिः कलिरद्य 'केरलवन' प्रायोऽब्दवृद्धत्वतः[1] ।।५६ ।।

(ग्रन्थप्रशस्तिः)

धरणिविधुबुधाच्छार्कारजीवार्किभानां
दिशति चरितबोधो भुक्तिमुक्तिं नृणां यत् ।
करणकुशलभावप्राप्तये प्रार्थयन्त्या-
मिह भवतु बुधानां मत्कृतौ कौतुकश्रीः ।।५७ ।।

लोकाद्यारामवासे त्रिदशमुनिसुसंघात[2] पद्योच्चभाढ्या
लोकाक्षिप्रीतिदा स्वर्णमयगुणयुता त्वत्पदालङ्कृतेयम् ।
लोकैः **सद्रत्नमाला** घनसुपरिमला धार्यते यैस्तु कण्ठे
'लोकाम्बे सिद्धसेव्ये' कलय सततमेतेषु सन्मङ्गलानि
।।५८ ।।

1. A_2 प्रान्तोऽब्दजातप्रिया । ; A_3 वृद्धत्वर

2. A_2 संख्यान

[1] इति श्रीमत् शङ्करवर्ममहाराजविरचितायाम् सद्रत्नमालायाम् षष्ठं गणितपरिष्करप्रकरणं समाप्तम् ।।

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1. A_1 एतच्छ्रीशङ्करवर्ममहाराजविरचितायां सद्रत्नमालायां पञ्चमप्रकरणं समाप्तम् ।।

$A_{2,4}$ इति सद्रत्नमालायां पञ्चमप्रकरणम् ।

A_3 श्रीकृष्णजयम् । इति सद्रत्नमालायां षष्ठप्रकरणम् ।

इति श्रीमच्छङ्करवर्ममहाराजविरचिता सद्रत्नमाला सम्पूर्णा ।

श्रीकृष्णजयम् ।; B_1 शुभमस्तु ।; A_4, $B_{2,3,4,}$ Mss. imcomplete and so no colopon.

Post colophonic statement :

A_3 यादृशं पुस्तकं दृष्ट्वा तादृशं लिखितं मया ।
शुद्धं वा यदि वाऽशुद्धं शोधयन्तु विपश्चितः ।।

Then is given the date of transcription in Kollam (Malayalam) era, which corresponds to A.D. 1846, Oct. 1.

कोल्लं २०२२ कन्निमासं १५ एळुतियतु । शुभमस्तु ।

Then are given the 24 divisor-sines for a place having latitude, *sāgara* (237) :

दिव्यो ननु भूपयानं सभारत्नं शमीवनम् ।
तुङ्गस्थानं कर्मीनृपः शूलि वन्द्यः शिवो दिव्याः ।।
नारी गोरी वन्द्यो हरिः मित्रे लोलः स्तम्भे जालम् ।
गोपो विद्वान् धूळी धावेत् पानेशोन्तः नृपो नेता ।।
प्राज्ञः शान्तः धीरो धाता सारङ्गोसौ धावेत् तीर्थम् ।
रागी धीस्थः रङ्गे पादं विद्वान् राजा धेनुं गर्जेत् ।।

इतु 'सागरं' (२३७) एन्नु पलाङ्गुलं उळ्ळटत्तेक्कु विलियादियायि उण्टाक्किय हारज्याक्कळ् । २४ ज्याक्कळ् उण्टु, ओरो ज्याविन्नुं ४-४ अक्षरसंख्या ।
हरिः श्रीगणपतये नमः । अविघ्नमस्तु ।

ENGLISH TRANSLATION

AND

NOTES

CHAPTER I

MATHEMATICAL OPERATIONS

1. Ever meditating on the lotus-feet of (Goddess) Śrī Pārvatī, the presiding deity of Lokamalayār temple, and also those of the teachers and paying obeisance to Sūrya and other (planets), I am writing this treatise (called *Sadratnamālā*) which is a compilation of the essence of the clear mathematical principles scattered in the ocean of astronomy, making it easy for those who wish to study the subject.

The Goddess Śrī Pārvatī, who presides over the Lokamalayār temple is the protecting deity of the principality of Kaḍattanāḍ. The author Śaṅkaravarman, a junior member of Kaḍattanāḍ Svarūpam, the royal family ruling the principality, first invokes the blessings of the protecting deity of his province. Lokamalayār temple is situated in Badagara (Lat. 11:36 N; Long. 75:35 E) on the west coast in Kerala state of India. The name of the temple is Lokamalayārkāvu in Malayalam, the native language of Kerala, and the Sanskritised term Lokāvanīdharasaridārāma is derived from the component Malayalam words *loka* (world) = *loka*, *mala* (mountain) = *avanidhara*, *ār* (river) = *sarit* and *kāvu* (garden) = *ārāma*.

The author then invokes the blessings of his teachers in order to guide him in using his knowledge, which they imparted to him, so that it could be passed on to the benefit of those who wish to study the subject of astronomy. He then pays obeisance to Sūrya, Candra, Kuja, Budha, Bṛhaspati, Śukra, Manda, Rāhu and Ketu who are the presiding dieties of the planets and nodal points viz. Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn, Ascending Node and Descending Node

respectively. Astronomy, which is a vast subject full of mathematical principles, is compared to an ocean containing various jewels. Like a string which binds together such jewels, this treatise brings together the mathematical principles relevant to astronomy in a single work and hence the title *Sadratnamālā* (A Garland of Good Jewels) is most appropriate.

2. I worship the great seers who, being repositories of unconditional kindness, are earthly Gods and by whose blessings I became full with good thoughts and devoid of evil.

The blessings of the great scholars are invoked in order to get inspiration and guidance for writing the treatise.

PATRONAGE

3. As per the orders of my (elder) brother, the Crown Prince Śrī Rāmavarman, the (younger) brother of the esteemed Ārya Udayavarman, who is the king of Bhaimībhūmi and who, being good-natured, shines like a pearl in Porlātiri family and who is (like) the (best) ornament of Kerala, I, Śaṅkaravarman, am writing this treatise for the pleasure of all those who know astronomy.

The author here refers to his patrons King Udayavarman and the Crown Prince Rāmavarman of Kaḍattanāḍ Kingdom, both of whom are his elder brothers. The work is undertaken as per the direct orders from the Crown Prince Rāmavarman. Kaḍattanāḍ is called the Land of Ghatoṭkaca, the son of Bhīma who is the second of the Pāṇḍava brothers of the epic *Mahābhārata*. Therefore, Kaḍattanāḍ is also called Bhaimībhūmi.

VASTNESS OF THE SUBJECT

4. Where is the subject of astronomy full of deep meaning? Where am I who is dull-witted? What is it that cannot be achieved by concentration (of the mind) on the lotus-feet of teachers?[1]

The vastness of astronomy full of deep meaning (in the sense that it is not easily comprehensible) and the limitation of the capacity of the author are mentioned here. In spite of this vast divergence, the author is confident that he will be successful in his endeavour with the blessings of his teachers.

DECIMAL SYSTEM OF NUMBERS

5-6 *ekam, daśa, śatam, sahasram, ayutam, niyutam, prayutam, koṭiḥ, arbudam, vṛndam, kharvaḥ, nikharvaḥ, mahāpadmaḥ, śaṅkuḥ, vāridhiḥ, antyam, madhyam* and *parārdham* are the names (in Sanskrit) of the numbers starting from unity to thousand crore crore.

The equivalents in mathematical symbols of the above Sanskrit terms are given below.

ekam	1
daśa	10
śatam	10^2
sahasram	10^3
ayutam	10^4
niyutam	10^5
prayutam	10^6

koṭiḥ	10^{7}
arbudam	10^{8}
vṛndam	10^{9}
kharvaḥ	10^{10}
nikharvaḥ	10^{11}
mahāpadmaḥ	10^{12}
śaṅkuḥ	10^{13}
vāridhiḥ	10^{14}
antyam	10^{15}
madhyam	10^{16}
parārdham	10^{17}

Ekam multiplied by ten is *daśa*, *daśa* multiplied by ten is *śatam*, *śatam* multiplied by ten is *sahasram* and so on. The numbers greater than thousand crore crore are not given separate names. It is to be noted here that the numbers obtained by dividing one by ten, hundred, thousand etc., are called *daśāmśa, śatāmśa, sahasrāmśa* etc., respectively of unity.

The reference to numbers in Vedic literature has to be made here. The following passage from *Vājasaneyasamhitā* (17.2) in *Śuklayajurveda* suggests the antiquity of the decimal of numbers.

इमा मे अग्न इष्टका धेनवः सन्त्वेका च दश च दश च शतं च शतं च सहस्रं च सहस्रं चायुतं चायुतं च नियुतं च नियुतं च प्रयुतं चार्बुदं च न्यर्बुदं च समुद्रश्च मध्यं चान्तश्च परार्धश्चैता मे अग्न इष्टका धेनवः सन्त्वमुत्रामुष्मिँल्लोके ॥

"O Agni, may these (sacrificial) bricks be mine. Own milch-kine: one and ten, a ten and hundred, a hundred and a thousand, a thousand and a ten thousand, a ten thousand and a hundred thousand, a hundred thousand and a million, a million

million or a billion. May these bricks be mine milch-kine in yonder world and this world."

There are other similar passages too.

MATHEMATICAL OPERATIONS

7. Addition, subtraction, multiplication, division and finding squares, square roots, cubes and cube roots of numbers constitute mathematical operations as stated by teachers (of mathematics).

In this stanza the author gives eight primary mathematical operations. The movements of celestial bodies can be studied by direct observation which also makes use of these mathematical operations and so a knowledge of them is essential.

ADDITION AND SUBTRACTION

8. Finding the sum of two quantities by adding the numbers in them either in direct order or in reverse order is called addition. (Similarly) finding the difference (between them) is subtraction.

The method of finding the sum of and the difference between two quantities, as given here, pertain to the procedure adopted when small objects like cowry shells or pebbles were used for calculations. In olden times, it was the usual practice to do mathematical calculations using cowry shells. A quantity is represented by placing cowry shells equal to the digits having the place values of unit, tens, hundreds etc. from right to left. The divisions of circle viz., *bhagaṇa* (a unit of 360 degrees of arc), *rāśi* (a unit of 30 degrees of arc), *bhāga* (degree), *kalā* (minute of arc), *vikalā* (second of arc), *tatparā* (1/60th) of a second), and *pratatparā* (1/3600th of a second) and the divisions of time viz. *abda* (year), *māsa* (month), *dina* (day), *ghaṭikā*

(1/60th of a day) *vighaṭikā* (1/3600th of a day), *prāṇa* (1/60th of a *vighaṭikā*) and *gurvakṣara* (1/60th of a *prāṇa*) are represented by placing the cowry shells from top to bottom.

The divisions of time are explained in the first stanza of second chapter and the divisions of circle are given in the second stanza of the same chapter.

As mentioned earlier, the divisions of circle are represented by placing the cowry shells from top to bottom denoting *bhagaṇa* to *pratatparā*. In the case of divisions of time they denote *abda* to *gurvakṣara*. The order from bottom to top is called the "direct order" and that from top to bottom is the "reverse" or "indirect order". The direct order, in the case of numbers, is from right to left in the order of increasing place values, i.e., unit, ten, hundred etc., and that from left to right is the reverse order. The numbers in the corresponding places are to be added either in the direct or in the reverse order to find the sum of two similar quantities. The same procedure is to be adopted for finding the difference between two similar quantities.

MULTIPLICATION

9. Multiply separately, the last, last but one etc. digits of the multiplicand by the multiplier. The sum of these (in accordance with the place values in the multiplicand) is the product. Or multiply, severally, the multiplicand by any number of terms into which the multiplier is split. The sum of these (also) is the product.

Two methods of finding the product of two quantities are given here. In the first method, the digit in the unit place, tenth place etc., of the multiplicand are separately multiplied by the

multiplier. Then add all these in accordance with their place value. The second method requires the multiplier to be split into any desired number of terms and the multiplicand to be multiplied by these terms. The sum of these gives the product.

Example

Consider an example in which x is the multiplicand and y is the multiplier. Let a, b, c etc., be digits having place values unity, ten, hundred etc. i.e.,

$$x = a + 10b + 100c + \ldots$$

The products of the digits a, b, c etc., with the multiplier y are ay, by, cy etc. The sum of these terms according to the place values of a, b, c etc., is $ay + 10by + 100cy + \ldots$ Thus the product

$$xy = ay + 10by + 100cy + \ldots$$

According to the second method, the multiplier is to be split into any desired number of terms. Let p, q, r . . ., be the terms into which the multiplier y is split. i.e., if

$$y = p + q + r + \ldots,$$

then the product $xy = xp + xq + xr + \ldots$

This is illustrative of the distributive property of multiplication over addition.

DIVISION

10. In division, the quotient is that which, on multiplication by the divisor, becomes equal to the dividend. Division by a divisor which is less than the dividend is carried out in the "reverse order" (of digits from left to right or from top to bottom as the case may be).

The first half of the stanza defines division and the second half gives the procedure. If a is the dividend and b the divisor the result c is given by

$$cb = a$$

The division is to be carried out from left to right in the numbers and from top to bottom in the case of divisions of circle as well as those of time in which they are to be multiplied by 12, 30, 60 etc., in appropriate places.

'In the reverse order' means that the operation has to be performed from the highest place. *Yuktibhāṣā* (p. 50) asserts that if the highest is hundredth place, the division has to be performed so that the product of the quotient and divisor is a multiple of hundred smaller than the given number. Then subtract the numbers and with the remainder proceed till the units place is reached. In the auto-commentary it is observed that the first result is on the right of the second, the second on the right of the third and so on, implying the same idea.

SQUARE

11. The product of two equal numbers is the square (of that number). The squares (of numbers from one to nine) are one, four, nine, sixteen, twenty-five, thirty-six, forty-nine, sixty-four and eighty-one in order.

After defining the square of a number, this stanza gives the squares of single digit numbers from one to nine. The numbers are denoted using the *Kaṭapayādi* system of notation, an explanation of which is given in stanza 3 of chapter 3.

12. Having placed the square of the last digit (in the line of the square), the remaining part, multiplied by twice the last digit, is added (on the right of the square

already placed). This procedure is repeated with the remaining digits (of the number).

Squaring a number of more than one digit is carried from left to right. The digit on the extreme left is called *antya* (the last) and that on its right is *upāntya* (the next to the last). *Antya* is squared and placed. The remaining part is multiplied from left to right by twice the last digit and placed as a part of the square already placed starting from the next place. The procedure is repeated until all the digits are finished.

Example

The following example illustrates the method. Suppose the square of 3456 is to be found out.

		3	4	5	6			
Place 3^2		9						
Add $2 \times 3 \times 4$		2	4					
Add $2 \times 3 \times 5$			3	0				
Add $2 \times 3 \times 6$				3	6			
	1	1	7	3	6			
Add 4^2			1	6				
	1	1	8	9	6			
Add $2 \times 4 \times 5$				4	0			
Add $2 \times 4 \times 6$					4	8		
	1	1	9	4	0	8		
Add 5^2					2	5		
Add $2 \times 5 \times 6$						6	0	
	1	1	9	4	3	9	0	
Add 6^2							3	6
	1	1	9	4	3	9	3	6

The squares of the digits are to be added to alternate places from left to right. These places are called *vargasthānas* (the square places). The places in between *vargasthāna* are called *avargasthānas* (the non-square places). Thus the squares are to be placed in the *vargasthāna* and the product of twice the last digits and the remaining parts are to be placed in the *avargasthāna*. It is interesting to see that the whole procedure is very handy when cowry shells (or any small objects like pebbles) are used for calculation instead of writing materials.

13. The product of two parts (into which a number is split), multiplied by two and added to the sum of the squares of the parts is the square (of that number). Or the sum of the square of any arbitrary number and the product of the sum and difference of the given number and the arbitrary number is (also) the square (of that number).

Two more methods of finding the square are given. In the first method, the given number is to be expressed as the sum of two parts. The sum of the squares of these parts to which twice the product of these parts is added, is the square. If a is the number, which is expressed as the sum of two numbers b and c, then

$$a^2 = b^2 + c^2 + 2bc$$

According to the second method, an arbitrary number is added to and subtracted from the given number and the product of these sum and difference is found. Add the square of arbitrary number to this product to get the square of the given number. If a is the given number and k is any arbitrary number, then

$$a^2 = (a + k)(a - k) + k^2$$

SQUARE ROOT

14. (Having deducted the maximum possible square from the last square place) divide the non-square place by twice the square root (of the maximum square earlier deducted). Deduct the square (of the quotient) from the next square place. Repeat this (to get the square root).

The number whose square root is to be determined, is denoted by placing the digits in a line. The odd places counted from right to left are the square places and the even places are the non-square places, as mentioned earlier. The maximum possible square (of numbers one to nine) is subtracted from the digit or digits in the last square place and keep the square root in a separate place. This is *prathamaphala* (the first result). Place the digit of the next non-square place on the right of the remainder and divide by twice the first result. This is *dvitīyaphala* (the second result). Place the digit of the next square place on the right of the remainder and deduct square of the quotient from it. Place the digit of the next non-square place on the right of the new remainder and divide by twice the second result. This is *tṛtīyaphala* (the third result). Place the digit of the next non-square place on the right of the remainder and divide by twice the third result. This is repeated until all the digits are exhausted. If the given number is not a perfect square, zeroes are placed in square and non-square places and the process is continued to any desired number of digits.

Example

Consider the following example in which the square root of 11943936 is to be found. The square places are marked *s* and the non-square ones *n*.

			s	*n*	*s*	*n*	*s*	*n*	*s*	
			11	9	4	3	9	3	6	(3456
Subtract 3^2			9							
Divide by 2×3	=	6)	2	9	(4					
			2	4						
				5	4					
Subtract 4^2				1	6					
Divide by 2×34	=	68)		3	8	3	(5			
				3	4	0				
					4	3	9			
Subtract 5^2						2	5			
Divide by 2×345	=	690)			4	1	4	3	(6	
					4	1	4	0		
								3	6	
Subtract 6^2								3	6	
								0		

Since the remainder is zero the given number is a perfect square and its square root is 3456.

CUBE

15. One, eight, twenty-seven, sixty-four, one hundred and twenty-five, two hundred and sixteen, three hundred and forty-three, five hundred and twelve, seven hundred and twenty-nine are the cubes of the numbers from one to nine. The product of three (equal numbers) is the cube (of that number).

16. To the cube (of the last digit) add (on the right) the product of thrice the square of the last digit and the remaining digits. Then add (on the next place to the right) the product of thrice the last digit and the square of the remaining part and then add the cube of the remaining part (on the next place to the right). This is (repeated until all the digits are finished to get) the cube.

Example

Consider an example in which the cube of 234 is to be found.

		2	3	4				
Place 2^3		8						
Add $3 \times 2^2 \times 34$		4	0	8				
	1	2	0	8				
Add $3 \times 2 \times 34^2$			6	9	3	6		
	1	2	7	7	3	6		
Add 3^3				2	7			
Add $3 \times 3 \times 4^2$				1	0	8		
Add $3 \times 3^2 \times 4$					1	4	4	
Add 4^3							6	4
	1	2	8	1	2	9	0	4

17. Having split the given number (whose cube is to be determined) into two parts, thrice their product is multiplied by each of them. Their sum to which the cubes of the parts are added is the cube (of the given number).

Let c be the given number which is expressed as the sum of two terms a and b. Then

$$c^3 = 3ab.a + 3ab.b + a^3 + b^3$$
$$= 3a^2b + 3ab^2 + a^3 + b^3$$

CUBE ROOT

18. (Having deducted the greatest possible cube from the last place and having kept the cube root of the number subtracted in the line of cube root), divide the second non-cube place by thrice the square of the cube root (and place the quotient on the right of the cube root kept earlier) and subtract the square of the quotient multiplied by thrice the cube root from the first non-cube place. Then subtract the cube (of the quotient) from the cube place. Repeat this until the digits are exhausted.

The places counted from right to left are called cube place, first non-cube place, second non-cube place, again cube place, first non-cube place, second non-cube place and so on.

Example

In the following example to illustrate the method given in the stanza, the cube places are marked by c and non-cube places are marked by n and n' respectively.

$$\begin{array}{ccccccc} c & n' & n & c & n' & n & c \\ 1 & 2 & 8 & 1 & 2 & 9 & 0 & 4 \end{array} \qquad \frac{234}{\text{Line of cube root}}$$

		1 2 8 1 2 9 0 4
Subtract 2^3		8
Divide by 3×2^2	=	12) 4 8 (3
		3 6
		1 2 1
Subtract $3 \times 2 \times 3^2$		5 4
		6 7 2
Subtract 3^3		2 7
Divide by 3×23^2	=	1587) 6 4 5 9 (4
		6 3 4 8
		1 1 1 0
Subtract $3 \times 23 \times 4^2$		1 1 0 4
Subtract 4^3		6 4
		6 4
		0

Thus the cube root is 234 as the remainder is zero.

Since the remainder is zero, the cube root is exact. It is to be noted that in step 2 the quotient is 3 and not 4 in order that the product of thrice the cube root and the square of the quotient can be subtracted from the next non-cube place.

ITERATIVE METHOD OF FINDING SQUARE ROOT AND CUBE ROOT

19. Divide the number (whose square root is to be found) by any arbitrary number and find half of the sum of the

arbitrary number and the quotient. Divide the number again by this new divisor. The same process is continued until the quotient becomes equal to the divisor. In the case of cube root, the first quotient is divided again by the arbitrary number to get the second quotient. The half of the sum of the arbitrary number and the second quotient is found which is the second divisor. This is repeated until the divisor becomes equal to the second divisor.[2]

<u>Example</u>

Let 625 be the number whose square root is required.

Divide by 10 (arbitrary number)		10)	6 2 5 (62
			6 2 0
Divide by (10 + 62)/2	=	36)	6 2 5 (17
			3 6
			2 6 5
			2 5 2
Divide by (36 + 17)/2	=	26)	6 2 5 (24
			5 2
			1 0 5
			1 0 4
Divide by (26 + 24)/2	=	25)	6 2 5 (25
			5 0
			1 2 5
			1 2 5
			0

Thus the square root is 25 since remainder is zero.

Let 512 be the number whose cube root is to be determined and let 10 be any arbitrary divisor.

Divide by 10	10) 5 1 2 (51
	5 1 0
Divide 51 by 10	10) 5 1 (5
	5 0
Divide 512 by (10 + 5)/2	7) 5 1 2 (73
	4 9
	2 2
	2 1
Divide 73 by 7	7) 7 3 (10
	7 0
Divide 512 by (7 + 10)/2	8) 5 1 2 (64
	4 8
	3 2
	3 2
Divide 64 by 8	8) 6 4 (8
	6 4
	0

As the first divisor and the second quotient have become equal and the remainder is zero, the cube root is 8.

NOTES

1. One is reminded of the verse, at the commencement of *Raghuvaṁśa* of Kālidāsa (I. 2):

   kva sūrya prabhavo vaṁśaḥ kva cālpaviṣayā matiḥ . . . |

   meaning, 'where is the dynasty of the Sun and where am I with poor intellect'.

2. See Appendix III for a mathematical justification of this procedure.

CHAPTER II

TERMINONLOGY

DIVISIONS OF TIME

1. Of (the divisions of time), *gurvakṣara, vighaṭikā, ghaṭikā* and *dina* (each, in order), sixty times the former is equal to the latter. Thirty times *dina* is *māsa* and twelve times *māsa* is a *sāvana abda.*

The divisions of time defined in this stanza are listed in the following table:

60 *gurvakṣaras*	=	1 *vighaṭikā*
60 *vighaṭikās*	=	1 *ghaṭikā*
60 *ghaṭikās*	=	1 *dina* (day)
30 *dinas*	=	1 *māsa* (month)
12 *māsas*	=	1 *sāvana abda* (year consisting of 360 days)

A day is divided into 60 *ghaṭikās* each of which is divided into 60 *vighaṭikās.* A *vighaṭikā* consists of 60 *gurvakṣaras.* The term *gurvakṣara,* literally meaning a long syllable, here signifies the time required to pronounce a long syllable. It is a unit of time which is 1 in 21600 parts of a civil day.

The units of time smaller than *gurvakṣara* and those greater than the *sāvana* year were in use. These units according to *Vaṭeśvara siddhānta* (I. 7-9) are given below :

Lotus-pricking time	=	1 *truṭi*
100 *truṭis*	=	1 *lava*
100 *lavas*	=	1 *nimeṣa* (twinkling of eye)
4½ *nimeṣas*	=	1 *gurvakṣara*
4 *gurvakṣaras*	=	1 *kāṣṭhā*

2½ *kāṣṭhās*	=	1 *prāṇa* (4 seconds)
6 *prāṇas*	=	1 *vighaṭikā*
60 *vighaṭikās*	=	1 *ghaṭikā*
60 *ghaṭikās*	=	1 *dina* (day)
30 *dinas*	=	1 *māsa* (month)
12 *māsas*	=	1 *sāvana abda* (year consisting of 360 days)
43,20,000 *sāvana abda*	=	1 *yuga*
72 *yugas*	=	1 *manvantara*
14 *manvantaras*	=	1 *kalpa*
2 *kalpas*	=	1 day of *Brahmā*
30 days of *Brahmā*	=	1 month of *Brahmā*
12 months of *Brahmā*	=	1 year of *Brahmā*
100 years of *Brahmā*	=	1 *mahākalpa*

2. The Celestial Circle rotates always (relative to earth) and completes one rotation in a time equal to 21600 *prāṇas*. This number (21600) is the (number of) minutes of arc in a circle. 6 *prāṇas* are (equal to) a sidereal *vināḍikā*.

A *prāṇa* is defined as the time equal to 1 in 21600 parts of the time taken for the rotation of the Celestial Circle relative to earth. This is same as the time taken by earth to rotate once about its own axis. It can be seen from the previous section that a *prāṇa* is equal to 10 *gurvakṣaras*. As the number of degrees of arc in a circle is 360, each of which is divided into 60 minutes of arc, there are 21600 minutes of arc in a circle. Therefore a *prāṇa* is the time taken by the earth to rotate through a minute of arc about its axis.

DIVISIONS OF THE CIRCLE

3. Sixty times *pratatparā* is *tatparā*. Similarly, (sixty times *tatparā*) is *viliptikā*. In the same manner, (sixty times *viliptikā*) is *kalā* and (sixty times *kalā*) is *lava*. This (*lava*) when multiplied by thirty is *rāśi* and *rāśi* multiplied by twelve is the *bhamaṇḍala*.

This stanza defines the angular measures in a circle. The following table gives the inter-relations of the divisions of circle:

60 *pratatparās*	=	1 *tatparā*
60 *tatparās*	=	1 *viliptikā* (second of arc)
60 *viliptikās*	=	1 *kalā* (minute of arc)
60 *kalās*	=	1 *lava* (degree of arc)
30 *lavas*	=	1 *rāśi*
12 *rāśis*	=	1 *bhamaṇḍala* (circle)

ASTERISMS

4. The Celestial Circle consists of 27 asterisms. A *rāśi* is two and a quarter asterism. There are 135 stellar *nāḍis* in a *rāśi*.

27 asterisms lying along the zodiac identify the Celestial Circle, which is divided into 12 *rāśis* each equal to 30 degrees. Therefore, a *rāśi* in terms of asterisms is 27/12 i.e., two and a quarter. An asterism is divided into sixty equal parts called stellar *nāḍis*. As 2¼ asterisms constitute a *rāśi*, the stellar *nāḍis* in a *rāśi* is $60 \times 2\frac{1}{4} = 135$. A circle contains 27×60 stellar *nāḍis* and therefore the stellar *nāḍis* contained in each degree is $27 \times 60/360 = 4\frac{1}{2}$. A stellar *nāḍi* contains 4½ stellar *vināḍis* and a stellar *vināḍī* contains 4½ stellar *gurvakṣaras*.

TITHI

5. There are thirty *tithis* in the (maximum) angular separation (possible) between the Sun and the Moon. A *rāśi* consists of two and a half *tithis* and there are one hundred and fifty *tithi nāḍis* in a *rāśi.*

The instant when the longitudes of the Moon and the Sun are equal along the same direction, with respect to earth marks the end of New Moon (*amāvāsyā*) and the beginning of *pratipat viz.*, the first day of the bright fortnight. Then the angular separation between the Sun and the Moon relative to earth is zero. The instant when the moon comes diametrically opposite to the Sun relative to the earth marks the end of Full Moon (*paurṇamāsī*). This angular separation between the Moon and the Sun relative to the earth is divided into 15 equal divisions each of 180/15 = 12 degrees called *tithi.* These 15 *tithis* are called *pratipat, dvitīyā, tṛtīyā, caturthī, pañcamī, ṣaṣthī, saptamī, aṣṭamī, navamī, daśamī, ekādaśī, dvadaśī, trayodaśī, caturdaśī* and *pañcadaśī,* meaning the first, the second and so on upto the fifteenth *tithi.* The dark fortnight also is divided into 15 *tithis* called *pratipat, dvitīyā* etc. as of bright fortnight. The fifteenth *tithi* of the bright fortnight is called *paurṇamāsī* and that of the dark fortnight *amāvāsyā.* Thus there are 30 *tithis* altogether in a complete circle of 360 degrees. A *rāśi,* therefore, contains 30/12 = 2½ *tithis* and hence there are 2½ × 60 = 150 *tithināḍis* in a *rāśi.*

PLANETS, RĀŚIS AND ASTERISMS

6. The Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn, the ascending node and descending node are the planets (*grahas*). *Meṣa* etc. are the *rāśis* and *Aśvinī* etc. are the asterisms.

The Moon, which is a satellite of the earth, is taken as a *graha*. The Nodes, which are the points of intersection of the ecliptic with the orbit of the moon, are also given the status of the *grahas*. The Moon and the Nodes move along the zodiac like the *grahas* and the mathematical principles behind their motion are the same though the latter always move in reverse direction along the zodiac.

The zodiac is divided into 12 equal divisions called *rāśis*. They, are *Meṣa, Vṛṣabha, Mithuna, Karkaṭaka, Simha, Kanyā, Tulā, Vṛścika, Dhanus, Makara, Kumbha* and *Mīna*.

Twenty seven asterisms into which the zodiac is divided are *Aśvini, Bharaṇī, Kṛttikā, Rohiṇī, Mṛgaśīrṣa, Ārdrā, Punarvasu, Puṣya, Āśleṣā, Maghā, Pūrva Phalgunī, Uttara Phalgunī, Hasta, Citrā, Svātī, Viśākhā, Anurādhā, Jyeṣṭhā, Mūlā, Pūrva Āṣāḍha, Uttara Āṣāḍha, Śravaṇa, Dhaniṣṭhā, Śatabhiṣak, Pūrva Bhādrapada, Uttara Bhādrapada* and *Revatī.*

DAYS, TITHIS, KARAṆAS AND YOGAS

7. The days (in a week) begin with Sunday and the *tithis* with *pratipat.* The *karaṇas* are *Kṛmi, Simha* etc. and the *yogas* are *Viṣkambha* etc.

The seven days in a week are *Sūrya Vāra* (Sunday), *Candra Vāra* (Monday), *Kuja Vāra* (Tuesday), *Budha Vāra* (Wednesday), *Guru Vāra* (Thursday) *Śukra Vāra* (Friday) and *Śani Vāra* (Saturday).

As mentioned earlier, the *tithis* are *pratipat* (first), *dvitīyā* (second), *tṛtīyā* (third), *caturthī* (fourth), *pañcamī* (fifth), *ṣaṣṭhī* (sixth), *saptamī* (seventh), *aṣṭamī* (eighth), *navamī* (ninth), *daśamī* (tenth), *ekādaśī* (eleventh), *dvadaśī* (twelfth), *trayodaśī* (thirteenth), *caturdaśī* (fourteenth) and *pañcadaśī* (fifteenth).

Karaṇa is half of a *tithi*. There are eleven *karaṇas*.

They are *Kṛmi* (worm), *Simha* (lion), *Vyāghra* (tiger), *Varāha* (pig), *Gardabha* (ass), *Gaja* (elephant), *Surabhi* (cow), *Viṣṭi* (a dog like animal), *Śakuni* (falcon), *Catuṣpāt* (quadruped) and *Sarpa* (snake). From the beginning of the second half of *pratipat* (first *tithi*) of the bright fortnight till the end of first half of *caturdaśī* (fourteenth *tithi*), the *karaṇas* repeat from *Simha* to *Viṣṭi* eight times in cyclic order. The second half of *caturdaśī* of dark fortnight is *Śakuni*. The first and second halves of New Moon are *Catuṣpāt* and *Sarpa* respectively. The *karaṇa* of the first half of *pratipat* of the bright fortnight is *Kṛmi*.

There are twenty seven *yogas* viz., *Viṣkambha, Prīti, Āyuṣmat, Saubhāgya, Śobhana, Atigaṇḍa, Sukarman, Dhṛtiḥ, Śūla, Gaṇḍa, Vṛddhiḥ, Dhruva, Vyāghāta, Harṣaṇa, Vajra, Siddhi, Vyatīpāta, Varīyān, Parigha, Śiva, Siddha, Sādhya, Śubha, Śubhra, Brahma, Mahendra* and *Vaidhṛti*.

The *Karaṇas* are also known by the names *Bava, Bāvala, Kaulava, Taitila, Garaja, Vaṇija* and *Bhadra*. These are known as *Carakaraṇas* and are distributed among the 56 half *tithis* starting from the second half of *śukla pratipat* to the first half of *kṛṣṇa caturdaśī*. The other *karaṇas* called *Sthirakaraṇas* corresponding to the second half of *kṛṣṇa caturdaśī* the first half of New Moon, the second half of New Moon and the first half of *śukla pratipat*. They are also known by the names *Śakuni, Catuṣpada, Nāgava* and *Kimstughna*.

Yoga is obtained by adding the longitudes of the Sun and the Moon. When the sum exceeds 360^0, deduct 360^0 from it. Thus there are 27 *yogas* corresponding to $0 - 13^0\ 20'$, $13^0\ 20' - 26^0\ 40'$, etc., each having a length of $13^0\ 20'$.

Karaṇa and *yoga* are purely of astrological significance.

MĀNDI NĀḌI

8. *Māndi Nāḍis* (at day time) for the days beginning from Sunday are (obtained) by reducing 30 repeatedly by 4. Those at night time are the same as those (at daytime) for the corresponding fifth day. This is accurate for (the places of) zero (terrestrial) latitude.

The rising time of *Māndi* (an imaginary planet) for places of Zero terrestrial latitude is given in this stanza. *Māndi* is supposed to rise at day time at 26, 22, 18, 14, 10, 6, and 2 *nāḍis* after Sun-rise on Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday respectively. At night time *Māndi* is supposed to rise at 10, 6, 2, 26, 22, 18, and 14 *nāḍis* after Sun-set on these days. The *nāḍis* given above are accurate for the places of zero terrestrial latitude. For other places they are to be calculated proportionately, as explained in Stanza 38, Chapter 4.

9. A day time consists of *prāhna, pūrvāhna, aparāhna* and *sāyāhna,* each of which is of six *ghaṭikās* duration for the places of zero (terrestrial) latitude.

The duration of day time for places of zero latitude is 30 *nāḍis* or *ghaṭikās.* This interval is divided into five equal parts. They are called *prāhna* (morning), *pūrvāhna* (forenoon), *madhyāhna* (noon), *aparāhna* (afternoon) and *sayāhna* (evening). Each of these intervals is of duration of six *nāḍis* for the places of zero latitude. For other places these intervals will decrease or increase in proportion with the duration of day time.

LINEAR MEASUREMENTS

10. One in eight thousand of a *yojana* is a *daṇḍa*. One fourth of this is a *kara*. One twenty fourth of this is an *aṅgula* and one sixtieth of this is to be remembered as a *vyaṅgula*.

The units for measuring length are defined. The table showing these units is given below:

60	*vyaṅgulas*	=	1 *aṅgula*
24	*aṅgulas*	=	1 *kara* (*hasta*)
4	*karas*	=	1 *daṇḍa*
8000	*daṇḍas*	=	1 *yojana*

Bhāskarācārya defines an *aṅgula* to be equal to eight *yavas* (barley corn) – '*yavodarairaṅgulamaṣṭasaṅkhyaiḥ*' (*Līlavatī*, v. 5).

Since the length of barley corn varies from place to place, it is not taken as a standard unit here.

The circumference of the equator has been taken as 3299 (*yojanas*) since the equatorial diameter is 7626 miles. Thus

$$\text{One } yojana = \frac{7626 \times 3.1416}{3299} = 7.2621 \text{ miles}$$

One *yojana* is approximately 7¼ miles according to this. The diameter of the earth is given as 1052 *yojanas* and this also agrees with the above.

The circumference of the orbit of the Moon is 2,16,000 *yojanas*. If the Moon's distance is taken as 2,40,000 miles, one *yojana* is about 3.5 miles.

There are situations suggesting that one *yojana* is about 16 miles.

MEASUREMENT OF VOLUME

11. *Kuḍuba, prastha, āḍhaka, droṇa, vaha* and *khārikā* are the units (of volume), increasing four times in order ending with a cubic *kara.*

The table showing the units of volume is given below:

4 *kuḍubas*	=	1 *prastha*
4 *prasthas*	=	1 *āḍhaka*
4 *āḍhakas*	=	1 *droṇa*
4 *droṇas*	=	1 *vaha*
4 *vahas*	=	1 *khārikā*

Khārikā, the largest of the above units, is equal to a cubic *kara*.

UNITS OF WEIGHT

12. One hundredth of *tulā* is a *pala* and one fourth of this is a *karṣa.* One sixteen of this a *māṣa.* One fifth of this a *guñjā.* Half of this a *yava.* Three *guñjās* make a *valla.*

The following table gives the units used for measuring weights:

2 *yavas*	=	1 *guñjā*
5 *guñjās*	=	1 *māṣa*
16 *māṣas*	=	1 *karṣa*
4 *karṣas*	=	1 *pala*
100 *palas*	=	1 *tulā*

3 *guñjās* make 1 *valla.*

COINS

13. One sixteenth of a *niṣka* is a *dramma*. A similar part of this is *paṇa*. One fourth of *paṇa* is *kākaṇī*. One twentieth of this is a *varāṭaka*.

The table of coins is as follows.

20 *varāṭakas*	=	1 *kākaṇī*
4 *kākaṇīs*	=	1 *paṇa*
16 *paṇas*	=	1 *dramma*
16 *drammas*	=	1 *niṣka*

DIRECTIONS

14. The *Yonis* (for the directions) beginning with east are *Dhvaja* (Flag), *Dhūma* (Smoke), *Simhā* (Lion), *Viṣṭi* (Dog), *Vṛṣa* (Bull), *Khara* (Donkey), *Ibha* (Elephant) and *Balibhuk* (Crow) in order. If the remainder obtained after dividing three times the perimeter of the house etc., measured in units of *hasta* and *aṅgula*, by eight, is odd, it is auspicious.

To find the *Yoni* of a building, measure its outer perimeter in units of *hasta* and *aṅgula* and multiply by three. The result is then divided by eight. Depending on whether the remainder is 1, 2, 3, 4, 5, 6, 7 or 8 the *Yonis*—*Dhvaja, Simhā, Vṛṣa* and *Ibha* are considered to be *Yonis* auspicious and the even *Yonis viz., Dhūma, Viṣṭi, Khara* and *Balibhuk* are considered to be inauspicious. In the case of courtyards, the inner perimeter is to be taken for the calculation of the *Yoni*.

The *Yoni* of buildings, courtyards etc. are given here as it is related to architecture which, in turn, requires mathematical calculations[1]. For this purpose, the perimeters of the different

fields such as square, rectangle, circle etc. are to be determined and therefore it can rightly be treated in a mathematical treatise like the present work.

NOTES

1. The method of finding *Yoni* using the perimeter of the house is described in works on architecture, Vāstu etc. See *Manuṣyālaya Candrikā*, 3.23.

CHAPTER III

THE ALMANAC

1. We prostrate before the Elephant-faced (Gaṇapati), Sarasvatī, Kṛṣṇa, Īśa's son (Subrahmaṇya), (Planetary deities) the Sun and others, teachers, Śrī Lokāmbā and Dakṣiṇāmūrti. Let their words of blessing confer prosperity on us.

The author begins this chapter paying obeisance to various deities. He prays to the Elephant-faced Gaṇapati, the remover of obstacles, Sarasvatī, the Goddess of speech, Kṛṣṇa, the protector of the world, deities of the planets Sun and others, teachers who imparted knowledge, Śrī Lokāmbā the Protecting Deity of Kaḍattanāḍ which is the native place of the author and Dakṣiṇāmūrti, the God of knowledge.

The line *gīr naḥ śreyaḥ* (may the words be for our prosperity) is the first of the famous *Candravākyas* (lunar mnemonics) of Vararuci, an ancient Indian astronomer. These *vākyas,* known as *Vararucivākyas,* give 248 daily longitudes of the Moon for 9 anomalistic months. By metaphorically including the first *Vararucivākya* in this stanza, the author shows respect to ancient astronomers and prays for inspiration.

RULE OF THREE

2. (Method of) obtaining the *icchāphala* with the *phala, icchā* and *pramāṇa* is called the 'Rule of Three'. The product of the *phala* and *icchā* divided by the *pramāṇa* gives the *icchāphala.*

To explain the terms involved, consider the following problem. Let *a* be the distance travelled by a body moving with a constant velocity in time *b*. It is required to find the distance in

another time *c*. Here *b* is the *pramāṇa* (the argument), *a* the *phala* (the fruit) and *c* the *icchā* (the requisition). The *icchāphala* (fruit corresponding to the requisition) *d* is obtained by the 'Rule of Three' as illustrated below.

$$icch\bar{a}phala = \frac{icch\bar{a} \times phala}{pram\bar{a}\d{n}a}$$

$$d = \frac{c \times a}{b} \qquad (3.1)$$

b and *c* are in units of time and *a* is in units of distance. Therefore *d* is also in units of distance. Thus *icchā* and *pramāṇa* will be in the same units while the unit of *icchāphala* will be the same as that of the *phala*.

KAṬAPAYĀDI NOTATION

3. The letters *na*, *ña* and the vowels (of Sanskrit alphabet) are (used to denote) zero. The numerals begin with *ka*, *ṭa*, *pa* and *ya*. In a conjunct (letter), the numeral is that of the consonant next to the last (letter). A consonant without a vowel (suffixed to it) is not to be considered (for denoting any numeral).

The following table gives the letters of Sanskrit alphabet and the numerals which they denote according to the *Kaṭapayādi* notation. The vowels suffixed to the consonants do not denote any numerals. They are suffixed only for the sake of pronunciation. The vowel *a* is suffixed here. Any other vowel may be used as well.

Consonants				Numerals
ka	*ṭa*	*pa*	*ya*	1
kha	*ṭha*	*pha*	*ra*	2

ga	*ḍa*	*ba*	*la*	3
gha	*ḍha*	*bha*	*va*	4
ṅa	*ṇa*	*ma*	*śa*	5
ca	*ta*		*ṣa*	6
cha	*tha*		*sa*	7
ja	*da*		*ha*	8
jha	*dha*		*ḷa*	9
ña	*na*			0

Vowels

a ā i ī u ū ṛ ṝ ḷ e ai o au 0

All vowels standing alone denote zero. In conjunct letter, the consonant next to the last letter is to be considered. Hence the stanza states *"miśre tu upāntya hal saṅkhyā"*.

Thus *dhī* (*dh* + *ī*) denotes 9, the numeral for the consonant *dh* which is next to the last letter *i*. Similarly *kti* (*k* + *t* + *i*) denotes 6 which is the numeral for *t*. *hṛt* (*h* + *ṛ* + *t*) is 8, the numeral for *h* which is the consonant with a vowel next to the last. The consonant *t* is not considered as it is not suffixed with a vowel.

Words denote the numbers with numerals written from right to left in the order of letters of words. Thus *kṣīrābdhiga* denotes 3926.

KOLAMBA, ŚAKA AND KALI YEARS

4. 3926 added to the elapsed Kolamba year or 3179 added to the elapsed *Śaka* year gives the corresponding elapsed Kali year, which is the number

of revolutions of the Mean Sun (completed since the beginning of Kali year).

The time taken by the earth to go once round the Sun is one year. This is the same as the time for relative 'motion of the Sun' along the ecliptic.

Kolamba year is also known as Kollam year.

5. Multiply 365 (days), 15 (*nāḍikās*), 31 (*vināḍikās*), 15 (*gurvakṣaras*) with Kali year and subtract 2 days, 8 (*nāḍikās*), 53 (*vināḍikās*), 14 (*gurvakṣaras*) from it. This is the number of days, *nāḍikas*, *vināḍikas* and *gurvakṣaras* elapsed since the beginning of the Kali Yuga, which started on a Friday. After subtracting 2 days from this, divide the result by 7 and the remainder is the *saṅkramaṇa dhruva* of the Sun.

The Kali year begins with *Meṣa* and the number of days, *nāḍikas*, *vināḍikas* and *gurvakṣaras* elapsed since the beginning of the Kali Yuga is calculated by the 'Rule of Three'. One year is the argument (*pramāṇa*) and the duration $365d$ $15n$ $31v$ $15g$ (*mukuṭolbaṇakṛṣṇatālaḥ*) is the fruit (*phala*). Elapsed Kali year is the requisition (*icchā*) and the corresponding *icchāphala*

$$= \frac{(\text{Kali year}) \times (365d\ \ 15n\ \ 31v\ \ 15g)}{1}$$

MEAN SUN

6. The number of days elapsed (since the beginning of the current month) is reduced by the (same) number (in *nāḍikās*) and increased by the same number divided by 7 (deg.) 30 (min.) and added to 28 (deg.) 22 (min.)

etc., for the months *Vṛṣabha* etc., to get the longitude of the Mean Sun at any instant.

The duration of a year as given in the previous stanza is $365d\ 15n\ 31v\ 15g$. The Sun completes one revolution relative to earth during this period. Therefore the angular distance traversed by the Sun per day is

$$= \frac{360^\circ}{365d\ 15n\ 31v\ 15g}$$

$$= 0^0\ 59'\ 8''\ \text{(corrected to a second)}$$

Therefore the angular distance traversed during N days is

$$= (0^0\ 59'\ 8'')\ N$$

$$= [0^0 + (60' - 1')60''/7.5]\ N$$

$$= [1^0 - 1' + 1'/7^0\ 30']\ N$$

$$= [N^0 - N' + N'/7^0\ 30']$$

This is to be added to the position of the Sun at the beginning of the month called the *dhruva* for the month. $28^0\ 22'$ is the *dhruva* for *Vṛṣabha.* The time interval in *nāḍikās* between the transit of the Sun to a *rāśi* and the Sunrise on the day of transit is to be added to or subtracted from the motion for the given days calculated as above depending on whether the former precedes or follows the latter.

The following table gives the *dhruvas* for all months.

Month	*Dhruva*			*Vākya*
Vṛṣabha	0	28	22	*śreṣṭhām hi ratnam*
Mithuna	1	29	19	*dhānya dharoyam*
Karkaṭaka	3	00	27	*sukhī anilaḥ*
Simha	4	01	29	*dharaṇyām nabhaḥ*

Kanyā	5	02	04	*vānarā amī*
Tulā	6	02	05	*munīndronantaḥ*
Vṛścika	7	01	33	*balāḍhya nāthaḥ*
Dhanus	8	00	38	*jale ninādaḥ*
Makara	8	29	35	*śūladharo hi*
Kumbha	9	28	37	*sāmbo hi pradhānaḥ*
Mīna	10	27	59	*dharmasukham nityam*
Meṣa	11	27	53	*laksmīḥ surapūjya*

The first column under *dhruva* represents *rāśi*, the second represents *bhāga* (degree of arc) and the third *kalā* (minute of arc).

AHARGAṆA

7. Add the elapsed Kali years to the Mean Sun at the Sunrise on (any) desired day and multiply it by 210389 and divide by 576. (The result is) the number of days elapsed since the beginning of the Kali yuga (till the given day).

The ratio 210389/576, which is equal to 365*d* 15*n* 31*v* 15*g*, is the number of days, *nāḍikās* and *vināḍikās* in a year. This, when multiplied by the elapsed Kali years, gives the number of days elapsed since the beginning of the current year till the desired day. The result obtained is divided by 7 and the remainder is counted from Friday. If the day arrived at does not agree, proper correction is made by adding or subtracting one or two days.

LONGITUDE OF THE SUN

8. Subtract 1773694 from the number of the Kali days (of any later date) and divide by 21185. From the remainder, subtract 116 (days) 2 (*nāḍikās*), 730 (days) 31 (*nāḍikās*) or 365 (days) 15 (*nāḍikās*) as the case

may be. The duration of the elapsed months is subtracted from this. The number of completed months is the *rāśi*, completed days the *bhāga* (degree) and the completed *nāḍikās* the *kalā* (minutes of arc). To this, *yogyādi* corrections are applied. This is the longitude of the Sun.

The day following the 1773694th Kali day, which corresponds to 10th April, 1755, is taken as the starting point. The number of days elapsed from this date is found out first and then divided by 21185 which is the number of days in 58 years.

The corrections for each month, in minutes of arc, are given for intervals of 8 days. So there are four values of corrections for each month. These values (in minutes of arc), called *yogyādi* corrections are given below.

Month	Corrections (in minutes)			
Meṣa	11 (*yogyā*)	14 (*vaidyaḥ*)	16 (*tapaḥ*)	17 (*satyam*)
Vṛṣabha	19 (*dhānyaḥ*)	21 (*putraḥ*)	22 (*kharo*)	24 (*varaḥ*)
Mithuna	24 (*vīraḥ*)	25 (*śūraḥ*)	25 (*śaro*)	24 (*vajrī*)
Karkaṭaka	24 (*bhadram*)	23 (*gotro*)	22 (*ruruḥ*)	21 (*karī*)
Simha	19 (*dhānyaḥ*)	17 (*sevyo*)	15 (*mayā*)	13 (*loke*)
Kanyā	11 (*kāyo*)	8 (*dīnaḥ*)	6 (*stanām*)	3 (*ganā*)
Tulā	1 (*yājño*)	1 (*yajñān*)	3 (*ganā*)	5 (*śuna*)
Vṛścika	6 (*tena*)	8 (*dīno*)	9 (*dhunīḥ*)	10 (*nataḥ*)
Dhanus	10 (*āpaḥ*)	11 (*pāpaḥ*)	11(*payaḥ*)	11 (*pathyam*)
Makara	11 (*pūjyo*)	9 (*dhenuḥ*)	8 (*dīno*)	7 (*ṛthinaḥ*)
Kumbha	6 (*tanuḥ*)	4 (*bhinnaḥ*)	4 (*ghanaḥ*)	0 (*jñānī*)
Mīna	2 (*ratnam*)	4 (*bhānuḥ*)	7 (*sunir*)	10 (*nayet*)

9. These numbers viz., 11 etc.,(*yogyā* etc.), are the corrections to *nāḍikās* for each month for intervals of 8 days.

The number of days in excess of multiples of eight is multiplied by the corresponding factor and divided by eight to get the correction for *nāḍikā* for the excess days.

10. Add the number of months and days elapsed to the time (in *nāḍikās*) of transit of the Sun to the current month. The correction (to be applied to this) is positive or negative for those beginning from *yajña* (1 min.) or *ratna* (2 min.).

The *yogyādi* corrections, listed above, are negative for the period from *Mīna* till 8th day of *Tulā* i.e., from *ratna* (2 min) to *yajña* (1 min) and positive after that till the end of *Kumbha* i.e., from *yajña* (1 min) to *jñanī* (0 min).

LONGITUDE OF THE MOON

11. Subtract 1794913 from the number of elapsed Kali days on any (later) day and divide by 12372. The remainder is divided by 3031 whose remainder is (again) divided by 248. This (final) remainder is the number of lunar mnemonics for the given day.

Lunar mnemonics are 248 [chronograms] which give the longitudes of the Moon for 248 days equal to 9 anomalistic months. These chronograms, popularly known as *Candravākyas*, give the daily longitude of the Moon as *rāśi*, degree and minute in *Kaṭapayādi* notation. This stanza gives the method of finding the serial number of the *Candravākya* corresponding to the given day.

12. Multiply 9*r* 27*b* 48*k* 09*v* 44*t*, 11*r* 07*b* 31*k* 10*v* 16*t* and 27*r* 43*b* 28*k* 39*v* respectively by the quotients (obtained as above) and add the results (in accordance

with their place values) to $6r\,28b\,25k\,56v\,41t$ to get the *dhruva* of the Moon for the places of zero meridian. Multiply the Moon's minimum motion by the *deśāntara* and add to the result obtained if the place is on the west of zero meridian (and subtract if the place is on the east). This is the Moon's *dhruva* at the place.

$9r\,27b\,48k\,9v\,44t$, $11r\,7b\,31k\,10v\,16t$ and $27b\,43k\,28v\,39t$, in which the symbols *r, b, k, v* and *t* denote the units *rāśi* (30 degrees of arc), *bhāga* (degree of arc), *kalā* (minute of arc, *viliptikā* (one sixtieth of a *kalā*) and *tatparā* (one sixtieth of a *viliptikā*) respectively, are to be multiplied respectively by the quotients obtained by dividing the days elapsed after the epoch by the divisors 12372, 3031 and 248 as described in the previous stanza. The sum of all these according to their place values is added to $6r\,28b\,25k\,56v\,41t$. This is the Moon's *dhruva* for the places of zero meridians. The mean motion of the Moon per day (viz.) $12^0\ 1'\ 50''$ (*eṇāṅkonusphuṭa*) is multiplied by the *deśāntara*, which is the difference in terrestrial longitudes of the given place and that of zero meridian expressed in minutes of arc. This is positive for the places lying on the west and negative for the places on the east of zero meridian.

13-14. Multiply one fifth of 67 by the longitudinal difference, expressed in seconds of arc, between the given place and the place of zero meridian. Multiply the last quotient (mentioned above) by 431 and the first quotient by 7 and find their sum. This is positive. Multiply the quotient obtained by dividing the days by 3031 with 106 and add the result to 1448. This is negative. Divide 48364 by the (algebraic) sum of the (positive and negative) results obtained above. This

is the *dhruva* divisor which is positive or negative depending on the sign (of the denominator).

In stanza 11, three divisors are given (*viz.*) 12372, 3031 and 248. The quotients obtained after dividing the days elapsed after the epoch by these divisors are used here as multipliers with positive sign for the first and the last and negative sign for the middle. The algebraic sum of these three results is the *dhruva* divisor with the sign of the resultant. For places on the east the sign is to be reversed.

15. Add the *dhruva* of the Moon to the *vākya* (for the Moon) for the given day and apply the corrections due to the duration of the day and (also) of the *dhruva.* This gives the longitude of the Moon at the Sunrise (at the given place). Half of the difference between the *vākyas* for the previous and the following days (of the given day) is the (lunar) motion at the sunrise for the day.

The *Candravākya* (lunar mnemonics) for any day is found by the method described in stanza 11 of this chapter. Suppose that the lunar mnemonics for any day is $24^0 09'$ (*dhenavaḥ śrīḥ*). The mnemonics for the previous day is $12^0 03'$ (*gīrnaḥ śreyaḥ*) and that for the following day is $1^r 06^0 22'$ (*rudrastu namyaḥ*). The motion of the Moon for the given day is

$$= [(1^r 06^0 22') - (12^0 03')] /2$$

$$= (24^0 19') /2$$

$$= 12 \text{ deg. } 9 \text{ min. } 30 \text{ sec.}$$

If the *vākya* for the given day is $12^0 03'$ (*gīrnaḥ śreyaḥ*) the motion of the Moon for the day is calculated by finding half of the difference between the *vākya* for the next day, (viz.) $24^0 09'$

(*dhenavaḥ śrīḥ*) and that for the previous day, which is zero. Thus the motion is

$$= (24^0\ 09')/2$$
$$= 12 \text{ deg. } 4 \text{ min. } 30 \text{ sec.}$$

If the *vākya* for the day is *bhavet sukham* (27^0 44′), the last *vākya*, the previous one *kaveḥ śakyam* (15^0 41′) is to be subtracted from it to get the motion. The following *vākya* is not taken, as it is zero at the initial position.

16. From the *Candravākya* (for any given day), subtract that for the previous day. This is the motion of the Moon at sunset (on the given day at the given place). Half of this is added to the sum of the *vākya* for the previous day and the *dhruva* (of the Moon). Apply the correction due to the duration of daytime and that due to the *dhruva*. The correction for one eighth (of the lunar motion) is then applied. This is the (longitude of the) Moon (at sunset).

17. Convert the lunar motion into minutes of arc and subtract 722 from it. Divide the result by the divisor of the *dhruva* correction. This is positive or negative depending on the sign of the divisor.

18. The difference between the lunar motions at the Sunset on any day and on the following day is (the correction) of one eighth of the lunar motion. This is greater than that for the following day. Otherwise it is negative.

The magnitude and sign of the correction for one eighth of lunar motion is to be determined by the procedure explained here. This is required for the calculations given in stanza 16.

LONGITUDE OF THE TITHI & YOGA

19. The longitude of the *tithi* is the longitude of the Moon from which the longitude of the Sun is subtracted. The longitude of the Moon to which the longitude of the Sun is added is the longitude of the *yoga*.

This stanza gives the methods of obtaining the longitudes of the *tithi* and the *yoga*. The *tithis* and the *yogas* are explained under stanza 7 of Chapter II.

QUARTERS OF NAKṢATRA & KARAṆAS

20-21. $3^0 20'$, $6^0 40'$, $10^0 0'$, $13^0 20'$, $16^0 40'$, $20^0 0'$, $23^0 20'$, $26^0 40'$ and $1^r 0^0 0'$ are the nine ends of quarters of the asterisms in a *rāśi*. Half of the *tithi* is the *karaṇa* (with end points at 6, 12, 18, 24, 30 etc).

Since a *rāśi* consists of two and a quarter asterisms, there are nine quarters of asterisms contained in a *rāśi*. The elongation of each quarter is (30/9) deg., which is equal to $3^0 20'$. All the 27 asterisms and the 11 *Karaṇas* are listed under stanza 6 of Chapter II.

22. $13^0 20'$ and 12^0 are multiplied by 1, 2, 3 etc. to get the longitudes of the end points of the asterisms and the *tithis*, beginning from *Dasra* (*Aśvini*) and *pratipat* respectively. Since each asterism is of $13^0 20'$, the end points of *Aśvini, Bharaṇi* etc., are obtained by multiplying $13^0 20'$ by 1, 2, etc. Similarly, since each *tithi* is of 12^0, the longitudes of the end points of *pratipat, dvitīya* etc., are obtained by multiplying 12 by 1, 2, etc.

23. The corrections for the motion of the Sun for eight days, which are *yajña* etc., (+ 1 etc.) and *ratna*

etc. (–2 etc.), are appropriately applied to 60 (minutes) to get the motion of the Sun. This, when added to the motion of the Moon, gives the motion of the *yoga* and when subtracted from the motion of the Moon gives the motion of the *tithi.*

The corrections to the Sun's motion for the intervals of 8 days, in minutes of arc, are given under stanza 8 of this chapter. These corrections are positive from the 9th day of *Tulā* (*yajña*) etc.) till the last day of *Kumbha* and negative from the 1st day of *Mīna* till the 8th day of *Tulā* (*ratna* etc.).

24. The motion (of a planet at any given day) is multiplied by the difference in days (between the given day and any other day) and subtracted from or added to the longitude (of the planet) depending on whether the day is earlier or later than the given day. For (planets having) retrograde motion, the correction is reverse.

The angular position of a planet at an earlier time from any given time is obtained by subtracting the motion for the period from the angular position at the given time. For a later time, it is to be added. If the planet has retrograde motion relative to earth, during the period, the motion for the period is to be added for earlier time and subtracted for later time. The planets other than the Sun and the Moon may have retrograde motion. The Nodes, on the other hand, always have retrograde motion.

25. The minutes of arc traversed and those to be traversed by the asterism, *tithi* and *yoga* divided by the (corresponding) extension in degree (give) the (time in) *nāḍikā* elapsed and that to be elapsed (by the asterism, *tithi* and *yoga* respectively).

Let x be the degree elapsed by an asterism, *tithi* or *yoga*. If s is the corresponding extension expressed in degree, then the time elapsed is

$$= 60\ (x/s)\ \textit{nāḍikās}$$

and

the time to be elapsed is

$$= 60\ (s - x)/s\ \textit{nāḍikās}$$

26. Half of (the degrees elapsed by) the asterism multiplied by nine (gives) the (time in) *nāḍikās* elapsed by the asterism. Half of (the degree elapsed by) the *tithi* multiplied by ten (gives) the time in *nāḍikās* elapsed by the *tithi*. The *nāḍikās* elapsed by the asterism multiplied by two and divided by nine (gives) the degrees elapsed by the asterism and the *nāḍikās* elapsed by the *tithi* divided by five (gives) the degrees (elapsed) in the *tithi*.

Each asterism is of $13^0\ 20'$ of arc, which corresponds to 60 *nāḍikās*. Therefore 1 deg. corresponds to

$$60/12 = 5\ \textit{nāḍikās}.$$

Thus the degree elapsed by the asterism, when multiplied by 9 and divided by 2, gives the stellar *nāḍikā*.

The length of each *tithi* is of 12^0 which corresponds to 60 *nāḍikās*. Therefore 1 deg. of a *tithi* corresponds to $60/12 = 5$ *nāḍikās*. Thus the degrees elapsed by the *tithi*, when multiplied by 10 and divided by 2, gives the *tithi nāḍikās*.

MOTION OF SUN IN A RĀŚI

27. The minutes of arc to be traversed and those traversed in a *rāśi* by the Sun are (separately) divided by (its)

own divisor. These are the *nāḍikās* of the *rāśi* after and before sunrise. The *nāḍikās* to be elapsed and those elapsed are calculated similarly from the (longitude of) the Sun to which six *rāśis* are added.

Let s be the length in degree of a *rāśi* at a place and x be the degrees traversed by the Sun in it. Then, the time elapsed in the *rāśi* before sunrise is

$$60\ (x/s)\ \textit{nāḍikās}.$$

The time to be elapsed by the *rāśi* after sunrise is

$$60\ (s-x)/s\ \textit{nāḍikās}$$

As the ascendant at the sunset is diametrically opposite to the Sun, the time elapsed and that to be elapsed before and after the sunset respectively, are obtained by doing similar calculations after adding six *rāśis* (i.e., 180 deg.) to the longitude of the Sun at sunset.

CHAPTER IV

ON JYĀS, ARCS AND OTHERS

CIRCUMFERENCE OF A CIRCLE

1. Multiply the square of the diameter of the circle by 12 and extract the square root. With this as the first term, form a series thus. To get the odd terms, divide the first term continuously by 9, and twice the numbers 1, 3, 5 . . . added to 1. To get the even terms, divide the first term by 3 and continuously by 9, and divide them by twice the numbers 2, 4 . . . minus 1. Subtract the sum of the even terms from the sum of the odd terms. The result is the circumference of the circle.

If D is the diameter of the circle, then the circumference, C is equal to $\sqrt{12}D\left(1-\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{5}\cdot\frac{1}{3^2}-\frac{1}{7}\cdot\frac{1}{3^4}\cdots\right)$. One can observe that this is equivalent to the result:

$$C=\sqrt{12}D\left(1-\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{5}\cdot\frac{1}{3^2}\ldots-\ldots\right)$$

$$=\sqrt{12}\,\sqrt{3}\,D\left(\frac{1}{\sqrt{3}}-\frac{1}{3}\cdot\frac{1}{\left(\sqrt{3}\right)^3}-\ldots\right)=\pi\,D$$

or $$\frac{\pi}{6}=\frac{1}{\sqrt{3}}-\frac{1}{3}\cdot\frac{1}{\left(\sqrt{3}\right)^3}+\frac{1}{5}\cdot\frac{1}{\left(\sqrt{3}\right)^5}-\ldots$$

2. Square the diameter of the circle, multiply by 12 and extract the square root. With this as the first term, develop the series thus: Divide this continuously by 3, and divide by 1, 3, 5, 7, 9, 11, . . . and form the terms. Add the odd terms and the even terms. Subtract the sum of the even terms from the sum of the odd terms. The result is the circumference of the circle. When this is done, the circumference of the big circle with the diameter equal to 10^{17} units is *bhadrāmbudhisiddha-janma-gaṇita-śraddhāsma-yad bhūpagīḥ* (by assignment of *Kaṭapayādi* numerals).

The series is the same as that in the first. The circumference of a circle with 10^{17} as diameter is given by 314 15926 5358 979324.

Thus this value of π is approximately 3.14159265358979324.

It is well known that the series called Leibnitz's power series for $\tan^{-1}x$ and the Gregory's series for π were known to the Kerala mathematicians of the medieval period. From the works on Astronomy written in Kerala, it is known that Mādhava of Saṅgamagrāma (14th Century A.D.) discovered them. The usual series for $\frac{\pi}{4}$ as given by Leibnitz is this:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} \ \ldots$$

A proof of this occurs in *Yuktibhāṣā* and has been studied by many. But a variant of this series is found in the above stanza. But there is also the series,

The arc $= \frac{sR}{c} - \frac{sR}{3c}.\frac{s^2}{c^2} + \frac{sR}{5c}.\frac{s^4}{c^4} - \ldots,$ s and c being R sine and R cosine respectively. (See stanza 10 supra).

These have been studied by C.T. Rajagopal, T.V. Vedamurthi Iyer, K.Mukunda Marar, M.S. Rangachari and others.[1]

Different authors here give approximation for π.

Nīlakaṇṭha Somayājin gives the following approximation for :

$$\pi = 3.141592653$$

Karaṇapaddhati (V.4) gives the following

$$\pi: 3.1415926536$$

The value given in v. 2 above in *Sadratnamālā* is a better approximation.

MEASURE OF LARGE ARCS

3. Divide a quadrant of the zodiacal circle into several equal parts. Then every large arc is obtained by adding the corresponding piece of arc to the previous large arc.

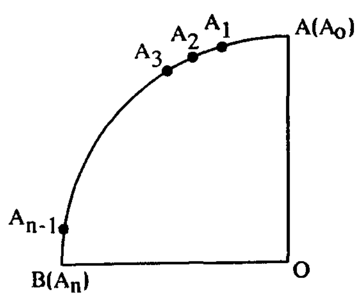


Figure 4.1

Let O be the centre of the circle. AB is an arc such that $\angle AOB = 90^0$. Divide the arc AB into the sub-arcs, $(A_0\ A_1)$, (AA_1), $(A_1\ A_2)$, . . . $(A_{n-1}\ A_n = A_{n-1}\ B)$. The arcs are called *cāpakhaṇḍas* (pieces of arc).

The arcs AA_1, AA_2, . . . , AA_n are called *mahācāpas*. It is asserted that every *mahācāpa* = the corresponding *cāpakhaṇḍa* + the previous *mahācāpa*. In the figure $AA_i = AA_{i-1} + A_{i-1}A_i$ for i=1, 2, . . . , n.

It is conventional to divide the arc in the quadrant into 24 equal parts of $3^0\,45'$ each. Āryabhaṭa (Gītikā, 7), Varāhamihira (*Pañcasiddhāntikā* IV.1), the author of *Sūryasiddhānta* (I.59) and many others do this way. But the radius of the circle is taken to be 120 by Varāhamihira. Āryabhaṭa takes the radius to be 3438′ which is equal to $\frac{180}{\pi} \times 60$ approximately. Significant improvements were made by Mādhava of Saṅgamagrāma who took the radius to be 3437′ 44″ 48‴ and Vaṭeśvara who divided the arc into 96 parts of 56′ 15″ each.

FINDING *JYĀ* AND ARC

4. Multiply the minutes of the circle (21,600) by 10^{17} and divide by the circumference. Then we get the diameter. Half of this is called *trijyā* or radius. The *jyā* of a *rāśi* (30°) is equal to half the radius.

$$\text{Diameter} = 10^{17} \times \frac{21{,}600}{\pi \times 10^{17}} = \frac{21{,}600}{\pi}$$

$$\text{Radius} = \frac{21{,}600}{2\pi} \cong 3438'$$

This is called *trijyā*.

It is necessary to explain the concept of *jyā* now.

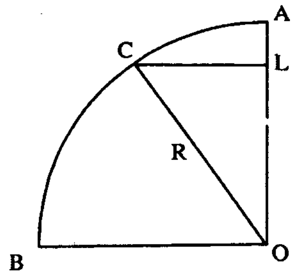


Figure 4.2

Let AB be the arc of a circle with centre at O and radius R. Let AC be an arc of length a with $\angle AOC = \theta$. Draw CL perpendicular to OA. Then CL is called *bhujajyā* of the arc AC and OL *koṭijyā* of the arc AC. AL is called *iṣu* or *bāṇa*. If the length of the arc AC is a, then we can write

bhujajyā a or *jyā* $a = R \sin \theta$ and

kotijyā $a = R \cos \theta$,

If θ is in radians, then $a = R\theta$. If

$$R = 3438' \cong \frac{180}{\pi} \times 60, \; a = R\theta = \frac{\theta}{\pi} \times 180 \times 60$$

Thus the arc a is only the angle in radians converted into minutes. The terms *bhujajyā* or R sine, *koṭijyā* or R cosine are used in this translation without difference.

5. Find the terms $\frac{(arc)^2}{2R}, \frac{(arc)^3}{2.3R}, \frac{(arc)^4}{2.3.4R}$, etc., R being the radius of the circle. Subtract the even terms continuously from the arc. The result is *bhujajyā*. The *koṭijyā* is obtained by subtracting the odd terms from R.

If a is the arc we get

$$\textit{bhujajyā} \text{ of } a = a - \frac{a^3}{3!R^2} + \frac{a^5}{5!R^4} \ldots$$

$$\textit{koṭijyā} \text{ of } a = R - \frac{a^2}{2!R} + \frac{a^4}{4!R^3} \ldots$$

The subtraction defined requires some explanation. If a, b, c are three numbers $a - b$ is obtained first. Then $b - c$ is found and $a - (b - c)$ is found ; $a - (b - c) = a - b + c$. Thus the terms are alternatively positive and negative.

These results are due to Mādhava of Saṅgamagrāma and given in *Tantrasaṅgraha* (p. 120), *Karaṇapaddhati* (VI. 10.13) and *Yuktibhāṣā* (pp. 91-9). A proof can be found in *Yuktibhāṣā* (pp. 160-94). The method uses the concept of *saṅkalita*, which can be considered as integration. The results are identical with the infinite series for sine and cosine obtained by Isaac Newton, in the west.

6. The square root of the difference between the square of the radius and the square of the *bhujajyā* is *koṭijyā*. By subtracting this from R, small *bāṇa* is obtained and by adding R to *koṭijyā*, large *bāṇa* is obtained. The product of the two *bāṇas* is *bhujajyā* squared. The assertion is

$$\sqrt{R^2 - R^2 \sin^2 \theta} = R \cos \theta$$

Now,

Large *bāṇa* $= R + R \cos \theta$

Small *bāṇa* $= R - R \cos \theta$

The product of the *bāṇas*

$$= (R + R\cos\theta)\,(R - R\cos\theta)$$

$$= R^2 - R^2\cos^2\theta = (R\sin\theta)^2$$

The result can be explained geometrically thus.

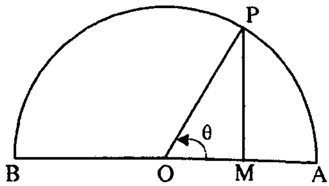


Figure 4.3

Consider a semicircle with extremities A and B. Let O be the mid point of AB and R be the radius.

Let P be a point on the semi circle such that $\angle AOP = \theta$

Then $R\sin\theta = PM$ and $R\cos\theta = OM$

Large *bāṇa* $= R + R\cos\theta = BM$ and

Small *bāṇa* $= R - R\cos\theta = MA$

Then large *bāṇa* × small *bāṇa*.

$$= BM\,.\,MA = PM^2 = (R\sin\theta)^2,$$

which is a geometrical result.

7. To find the *bhujajyā* of the sum or difference of arcs, multiply the *bhujajyā* of the first by the *koṭijyā* of the second and then multiply the *koṭijyā* of the first by the *bhujajyā* of the second. Add the two products so formed or subtract the second from the first according as *bhujajyā* of the sum or difference is required, and divide by the radius.

Let the arcs be equal to a and b respectively. Then the above rule gives $R\sin(a \pm b) = \dfrac{R^2\sin a\cos b \pm R^2\cos a\sin b}{R}$.

This is equivalent to the result $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$.

The rule given in the stanza is called *jīveparasparanyāya* and a proof is found in *Yuktibhāṣā* (pp. 91-9).

8. The *utkramajyās* are obtained by subtracting the first *jyā* from *R*, the second *jyā* from *R* etc. and so on at equal intervals and writing them in the reverse order.

The term *utkramajyā* stands for *bāṇa* or *iṣu* or *R* versed sine. In other words *utkramajyā* $a = R(1 - \cos a)$. The above rule states that

$$R(1 - \cos a) = R[1 - \sin(90^0 - a)].$$

The *utkramajyā* for $3^0\ 45' = R - R \sin 86^0 15' = 3438' - 3431' = 7'$.

9. Astronomers know that the diameter of the circle is obtained by adding to *bāṇa*, the quantity got by dividing the square of the *bhujajyā* by *bāṇa*.

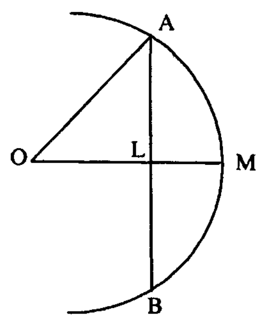


Figure 4.4

The picture of a bow or *cāpa*, chord called *guṇa, jīva* or *jyā* and arrow called *bāṇa* or *iṣu* is shown here.

Let arc $AM = a$ so that $\angle AOM = a$. Let $OA = R$.

Then $AL = R \sin a$. This is called *jyārdha* in the text, because the *jyā* is actually AB for the arc AMB. Considering the large *bāṇa*, we get

$$bāṇa + \frac{(bhujajyā)^2}{bāṇa}$$

$$= R\,(1 + \cos a) + \frac{R^2 \sin^2 a}{R(1 + \cos a)}$$

$$= 2R\cos^2 a/2 + \frac{4R^2 \sin^2 a/2\ \cos^2 a/2}{2R\ \cos^2 a/2}$$

$= 2R(\cos^2 a/2 + \sin^2 a/2) = 2R =$ the diameter of the circle.

The result can be proved, using the small *bāṇa* also.

10. The arc corresponding to a given *bhujajyā* is obtained thus. Multiply the radius by *bhujajyā* and divide by *koṭijyā*. This is the first result. Multiply this by the square of the *bhujajyā* and divide by the square of the *koṭijyā*. Repeat the process and divide the results by 1, 3, 5, ... Subtract the sum of the even results from the sum of the odd results.

If θ is the angle subtended by the arc in radians, the length of the arc $R\theta$ is the angle expressed in minutes.Thus we get

$$R\theta = R\frac{R\sin\theta}{R\cos\theta} - \frac{1}{3} \cdot R\frac{R^2\sin^3\theta}{R^2\cos^3\theta} + \ldots$$

$$= R\tan\theta - \frac{1}{3}R\tan^3\theta + \ldots$$

By the earlier observations, $R\theta$ is the angle in minutes. Thus

$$\theta = \tan\theta - \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} + \ldots$$

if θ is in radians. This is the well known series attributed to Gregory in the west.

11. When the arc is small, find the *bhujajyā* by subtracting from it the cube of the arc divided by six times the square of the radius. For getting the arc from the *bhujajyā*, add to it the cube of the *bhujajyā* divided by 6 times the square of the radius. This process can be carried on more than once.

We get $R\sin\theta = R\theta - \frac{R^3\theta^3}{6R^2} = a - \frac{a^3}{6R^2}$ where a is small by applying the rule in verse 5 above omitting further terms since the arc is small.

Also $a = R\theta$

$$= R\sin\theta + \frac{R^3\sin^3\theta}{6R^2}$$

This is the first approximation. This value of $R\theta$ can be substituted in the equation

$$R\theta = R\sin\theta + \frac{R^3\theta^3}{6R^2}$$

and a better approximation can be obtained. The method can be applied successively. If the approximation is to be correct up to minutes $\frac{(R\theta)^5}{5!R^4} < \frac{1}{2}$ (by the principle of rounding off)

i.e. $R\theta < \left(\frac{1}{2}5!R^4\right)^{\frac{1}{5}} = \left(60R^4\right)^{\frac{1}{5}} = 1530' = 25^0 30'$

for the choice $R = 3438'$

Thus the approximation can be used up to $25^0 30'$

If it needs to be correct up to seconds, we get

$$R\theta < \frac{\left(60R^4\right)^{\frac{1}{5}}}{60^{\frac{1}{5}}} = \left(R^4\right)^{\frac{1}{5}} = 674'$$

$$= 11^0 14'$$

For the same choice of R, the formula can be used up to arc of this length to get accuracy up to seconds.

12. Multiply by 1, 2, etc. the square of *trijyā*, and divide by 10. Extract the cube root and subtract 1, 2, . . . seconds. Then *jyās* corresponding to the arcs diminished by these are obtained.

We have, for any x in radians, Rx is in minutes of angle

$$R\sin x = Rx - \frac{R^3x^3}{6R^2} \text{ approximately.}$$

If $Rx - R\sin x = 1''$, then

$$\frac{1}{60} = \frac{R^3x^3}{6R^2}$$

Therefore, $R^3x^3 = \dfrac{6R^2}{60} = \dfrac{R^2}{10} \times 1.$

Extracting cube roots on either side,

$$Rx = \sqrt[3]{\frac{R^2 \times 1}{10}}$$

Then, $R \sin x = \sqrt[3]{\frac{R^2 \times 1}{10}} - 1$

In this way one can calculate the *jyās*. These are called *gūḍhamenakādijyās.*

They are

gūḍhamenakā	105′ – 43″
pūjyo gāṅgeyaḥ	133′ – 11″
candraśrīmayaḥ	152′ – 26″
stambhasthitikṛt	167′ – 46″
gūḍhohnidīpaḥ	180′ – 43″ etc.

When the arc x is small $R \sin x$ is almost equal to Rx in minutes. But more accuracy is achieved in the method given above. Thus *gūḍhamenakā* or 105′ 43″ is the *jyā* of the arc 105′ 44″. In the next *jyā* the difference is 2″. Thus *pujyo gāṅgeyaḥ* or 133′ 11″ is the *jyā* of 133′ 13″. In the next, the difference is 3″. Consequently *candraśrīmayaḥ* or 152′ 26″ is *jyā* for 152′ 29″. The 24th *jyā* is *tilāṅgho nīlaḥ.* This represents 306′ 36″ and is the *jyā* of 306′ 36″ + 24″ = 307′.

It is to be noted that even four figure tables give the values of trigonometrical ratios only up to 6′ and with differences up to 1′. But greater accuracy is achieved in the method given in the stanza. Moreover *gūḍhamenakādi vākyas* can be used to those, which differ by a few minutes.

KENDRA AND *PĀDA* (QUADRANT) DEFINED

13. The mean longitude, the true longitude itself or the true longitudes increased by three *rāśis*, six *rāśis*,

ayanāmśa etc., are called *kendra* when *mandocca, śīghrocca, pāta* etc. are subtracted. At times, half the quantities are considered.

Kendra is a general term. When the *mandocca* is subtracted from the mean longitude, *mandakendra* is obtained. When *śīghrocca* is subtracted from mean longitude it is called *śīghrakendra* and so on. The terms '*ādi*' is to show that the term is used in general context.

14. The six *rāśis* from *Meṣa* are called *Meṣādi* or northern and the six *rāśis* from *Tulā* are called *Tulādi* or southern. The *jyā* for the *kendra* has to be subtracted if it is *Tulādi* and added if it is *Meṣādi*. Sometimes, these have to be reversed.

Let x be the *Kendra*, i.e. the mean longitude, true longitude or the quantity after subtracting *mandocca*, *śīghrocca* etc. If $0 \leq x \leq 180^0$, it is called *Meṣādi* and if $180^0 \leq x \leq 360^0$, it is called *Tulādi*. For example, if $m = 200^0\ 4'$ is the mean longitude of the moon and the *mandocca* is $80^0\ 2'$ the *mandakendra* = $200^0\ 4' - 80^0\ 2' = 120^0\ 2'$.

This is *Meṣādi* and therefore the *mandaphala* has to be subtracted.

15. The arc from 0^0 to 90^0 is called odd (*oja*), that from 90^0 to 180^0 is called even (*yugma*), that from 180^0 to 270^0 is called odd (*oja*) and that from 270^0 to 360^0 is called even. In an arc of 90^0, the initial part is called *bhujā* and its complement is called *koṭi*. In the first quadrant the arc is called *bhujā* and in the second, *koṭi*, in the third *bhujā* and in fourth, *koṭi*.

In Indian Mathematics the *R* sine or *R* cosine is found, reducing the arc to the first quadrant. The sign is decided according to the context. Since sine is positive in the first two quadrants and negative in remaining quadrants, the first is called *Meṣādi* and the second *Tulādi.* In the case of *R* cosine, it is positive in the 1st and 4th quadrants and negative in the remaining. So *Makarādi* and *Karkyādi* are introduced. If $90^0 \le x \le 270^0$, it is called *Karkyādi* and if $270^0 \le x \le 360^0$, it is called *Makarādi.*

16. When the *kendra* does not exceed 90^0, its *jyā* can be known (from the *jyā*-table). When it lies between 90^0 and 180^0, subtract from 180^0 and find the *jyā.* When it lies between 180^0 and 270^0, subtract 180^0 and find the *jyā.* When it lies between 270^0 and 360^0, subtract from 360^0 and find the *jyā.*

The method of finding R sine is given above. The method is to reduce it to the first quadrant, find the *jyā*, and assign the positive or negative according as it is *Tulādi* (*Meṣādi*) or *Meṣādi* (*Tulādi*). It simply uses the fact that

$$\sin(180^\circ - \theta) = \sin\theta, \sin(180^\circ + \theta) = -\sin\theta \text{ and } \sin(360^\circ - \theta) = -\sin\theta.$$

Koṭiphala is positive if *Makarādi* and negative if *Karkyādi.*

FINDING THE *BHUJAJYĀ* OF AN ARC AND THE ARC OF A GIVEN *BHUJAJYĀ*

17. To find the *bhujajyā* of an arc, find from the table of *jyās* the arc near the given arc, greater or less. Find the difference of the two, divide by the diameter, multiply by 2 and *koṭijyā.* Then add this to the *bhujajyā* of the near arc or subtract from it according as it is less than or greater than the given arc.

We have $\sin(\theta + h) = \sin\theta \cos h + \cos\theta \sin h$

$$= \sin\theta + \cos\theta . h,$$

if θ and h are in radians and h is small.

Then , $R\sin(\theta + h) = R\sin\theta + (R\cos\theta)h$

$$= R\sin\theta + \frac{(R\cos\theta)(Rh)}{R}$$

$$= R\sin\theta + \frac{2(R\cos\theta)(Rh)}{2R}$$

This is the rule given in the stanza. Since $R = 3438' = \frac{180°}{\pi}$ the effect of multiplying by R is to convert radians to minutes.

18. Find the sum of the *koṭijyās* of the arcs and divide by the difference of the neighbouring *bhujajyās* (*samīpatajjyayoḥ*)^2. Divide $2R$ by this quantity to get the difference of arcs.

Let θ, and $\theta + h$ be the lengths of the arcs corresponding to neighbouring *bhujajyās*. We get:

$$\frac{R[\cos(\theta + h) + \cos\theta]}{R[\sin(\theta + h) - \sin\theta]}$$

$$= \frac{2\cos(\theta + h/2)\cos h/2]}{2\cos(\theta + h/2)\sin h/2}$$

$$= \frac{\cos h/2}{\sin h/2} = \frac{1}{h/2}, \text{ (if } h \text{ is in radians and is small)},$$

$$= \frac{2}{h}$$

Then, $\dfrac{2R}{\frac{2}{h}} = Rh$ is the angle in minutes.

MAXIMUM DECLINATION

19. The enlightened say that the *bhujajyā* of the maximum declination is given by that of 24^0. From practical experience it is observed that it undergoes a reduction by $32'$.

The declination of the Sun is maximum when it is equal to the obliquity, which is approximately equal to 24^0. A more accurate value is $23^0\ 28'$ which leads to a value of obliquity less by $32'$.

FINDING THE *JYĀ* OF ANY DECLINATION

20. The maximum declination multiplied by the *bhujajyā* of the *sāyana* longitude (of the sun) and divided by R gives *krānti* or declination. The *koṭijyā* of the declination is *dyujyā* and when it is subtracted from R, the *apamabāṇa* is obtained.

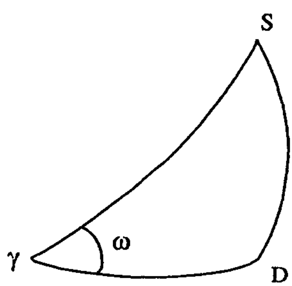


Figure 4.5

S is the position of the Sun on the ecliptic, SD the declination circle, D being the foot of the declination circle on the equator γD. SD = the declination of the Sun = δ. Let $\gamma S = \ell$ = the longitude (*sāyana*) of the Sun, $\angle S\gamma D = \omega$, the obliquity. We get

$$\sin\omega = \frac{\sin SD}{\sin\gamma S} = \frac{\sin\delta}{\sin\ell}$$

Therefore $\sin\delta = \sin\omega \sin\ell$. In the rule given, δ and ω are treated as small quantities. Consequently,

$$\delta = \omega \times \sin\ \ell$$

$$= \omega \times \left(\frac{R\sin\ell}{R}\right) = (R\sin\ell) \times \frac{\omega}{R}$$

In the auto commentary it is mentioned that *krāntijyā* is obtained by multiplying the *jyā* of the *sāyana* longitude by 5593 (*gaḷamarma*) and dividing by 13751 (*kṛṣṇasallāpa*). Then

$$\delta = R \times \sin\ell \times \frac{5593}{13751}$$

We have $\sin\delta = \sin\ell \times \sin\omega$. The result is more accurate, if $\omega = 23^0\ 28'$ and $R = 3438'$, we get :

$$\frac{\omega}{R} = \frac{23°18'}{3438'} = \frac{1408}{3438} = \frac{5632}{13752} \cong \frac{5593}{13751}$$

PRĀṆAKALĀNTARA

21. Find the product of the *bhujajyā, koṭijyā* of the *sāyana* longitude of the Sun, and the maximum *apamabāṇa*. Divide by R and *dyujyā*. The result is *prāṇakalāntara*. It is positive in the even quadrants and negative in the odd quadrants.

prāṇakalāntra = |Right Ascension – longitude|

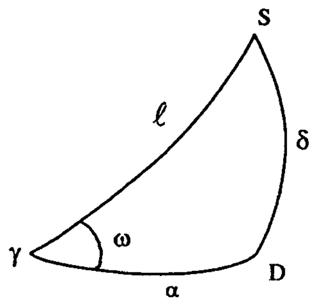


Figure 4.6

Let S be the position of the Sun on the ecliptic, SD the declination circle, D being the foot of the declination circle on the equator γD.

Let the longitude $= \gamma S = \ell$,

Right Ascension $= \alpha = \gamma D$,

obliquity $= \angle S\gamma D = \omega$.

We get

$$\sin\omega = \frac{\sin\delta}{\sin\ell}, \text{ and therefore,}$$

$$\tan\omega = \frac{\tan\delta}{\sin\alpha}$$

$$\sin\alpha = \frac{\tan\delta}{\tan\omega} = \frac{\sin\delta\cos\omega}{\cos\delta\sin\omega} = \frac{\sin\delta\cos\omega}{\cos\delta\sin\delta}\sin\ell$$

$$= \frac{\cos\omega\sin\ell}{\cos\delta}$$

Also

$$\cos\alpha = \frac{\cos\ell}{\cos\delta}$$

Therefore,

$$\sin(\ell-\alpha) = \sin\ell\cos\alpha - \cos\ell\sin\alpha$$

$$= \frac{\sin\ell\cos\ell}{\cos\delta} - \frac{\cos\ell\cos\omega\sin\ell}{\cos\delta}$$

$$= \frac{\sin\ell\cos\ell(1-\cos\omega)}{\cos\delta}$$

or,

$$R\sin(\ell-\alpha) = \frac{R\sin\ell\cos\ell(1-\cos\omega)}{cos\delta}$$

$$= \frac{(R\sin\ell)(R\cos\ell)R(1-\cos\omega)}{R.R.\ cos\delta}$$

This is the rule given in the stanza. Since the angle is small it is taken as *prāṇakalāntara* instead of *prāṇakalāntarajyā.*

The correction has to be made in the longitude to get the Right Ascension. Thus it is negative in the odd quadrants and positive in the even quadrants. One can easily verify that when $0 < \ell < 90°$, $\delta > 0$, $\sin\ell > 0$, $\cos\ell > 0$. Thus $\ell - \alpha > 0$ and $\ell > \alpha$. The correction is negative. In the second quadrant, $\sin\ell > 0, \cos\ell < 0, \cos\delta > 0, \ell - \alpha < 0$ and the correction is positive. In the auto commentary the value of $R(1-\cos\omega)$ is given as *gopasindhūram* i.e. 297′ 13″.

CONSTRUCTION OF FIELDS

22. A triangle and a quadrilateral can be constructed using one hypotenuse or two diagonals as the case may be. A circle can be constructed with *karkaṭayantra,* and also with the help of a string attached to a point. All these things are to be done on plain ground resembling the area filled with still water.

It is said in the commentary that one gets a *lambasūtra* by attaching a string to material made of copper or stone firmly to the earth[3].

ŚAṄKU AND THE DETERMINATION OF THE NORTH-SOUTH LINE

23. The *śaṅku* consists of a heavy cylindrical rod of base diameter 2 *aṅgulas* and height 12 *aṅgulas.* A needle of 12 *aṅgulas* is to be fixed at the centre of the top and with this, its height is equal to one *hasta* or 24 *aṅgulas.*

Śaṅku is a stylus (gnomon) that is planted on the level ground and it is of fundamental importance in astronomy. In the auto commentary it is said that it can be made of bull's horn, ivory etc. Description of the *śaṅku* varies from text to text though every one agrees that a rod should be vertically planted.

24. Note the points at which the tip of the shadow of the *śaṅku* meets the circumference of the circle before and after noon. This gives (on joining) the east-west line. Draw the circles with these as centres. The line joining the points of intersection is the North-South line.

After fixing the gnomon, the procedure for fixing the directions is given above.

AYANĀMŚA

25. Divide the Kali year by 615. This gives *rāśi* etc. Find the *bhujajyā* of the declination corresponding to this and then the arc. This gives *ayanāmśa* with the appropriate sign, in *Parahita* system.

We can consider the Kali year 2460. Dividing by 615, we get 4 *rāśis*, which is equal to 120^0. The *bhujā* is 60^0. The sign is negative, being *Meṣādi* and

ayanāmśa = $\sin^{-1}(\sin 60^0 \sin 24^0) = 20^0\ 59'$.

26. Divide the Kali year by 600. This gives the *rāśi* etc., of *ayanāmśa* in *Dṛk*. For the first 600 years it is 10^0, for 1200 years it is 18^0 and for 1800 years, it is 27^0.

According to the modern theory, the first point of Aries moves westwards at the rate of about 50.2″ per annum completing one revolution in 21,600 years. But *Sūryasiddhānta* (III.10-11a) gives a theory of libration according to which the first point of Aries oscillates about the point *Meṣādi* (the sidereal first point of the ecliptic), in 5400 years. A similar theory is followed in the text. This topic is discussed in detail in the book *Muddle of Ayanāmśa.*[4]

PALĀṄGULA

27. The *palabhā* is the length of the shadow at noon on the day when the *sāyana* Sun is at the end of the zodiac, in *aṅgulas* and *vyaṅgulas*. At Lokamalayārkāvu it is 2 – 28 (*hariśrīḥ*).

The length of the shadow of the *śaṅku* at midday on the day when the *sāyana* Sun is at the end of the zodiac (the equinoctial day) expressed in *aṅgulas* and *vyaṅgulas* is called *śaṅkucchāyā* or *palabhā*. At the place concerned (Lokamalayārkāvu) it is *hariśrīḥ* or 2 *aṅgulas* and 28 *vyaṅgulas*. (1 *aṅgula* = 60 *vyaṅgulas*).

If φ is the latitude of the place and h, the height of the *śaṅku*, then

$$palabh\bar{a} = h \tan\varphi$$

If $h\tan\varphi = 12\tan\varphi = 2^a\,28^v$, then $\tan\varphi = \dfrac{2^a\,28^v}{12} = \dfrac{148}{720} = 0.2006$

Therefore, the latitude of Lokamalayārkāvu $\varphi = 11^0\,20'$,

The auto-commentary gives the *palāṅgula* for various places. The values are given below:

Place	*palāṅgula*	Equivalent latitude	Modern figure for this latitude
Place of latitude 0	0	0	0
Near Thiruvananthapuram	*śivāya* ($1^a\,45^v$)	$8^0\,18'$	$8^0\,31'$
Near Kollam	*vāṇijyā* ($1^a\,54^v$)	9^0	$8^0\,31'$
Thiruvalla	*āgnīndra* ($2^a\,0^v$)	$9^0\,27'$	$9^0\,20'$
Kotungallur	*dhanendra* ($2^a\,9^v$)	$9^0\,39'$	$10^0\,05'$
Peruvanam	*rājyaśrī* ($2^a\,12^v$)	$10^0\,23'$	- -

Place	*palāṅgula*	Equivalent latitude	Modern figure for this latitude
Sivapuram (Trichur)	*Gopura* ($2^{a}\ 13^{v}$)	$10^0\ 28'$	$10^0\ 30'$
Alathur	*duṣkara* ($2^{a}\ 18^{v}$)	$10^0\ 51'$	$10^0\ 55'$
Kozhikode	*śrīrudra* ($2^{a}\ 22^{v}$)	$11^0\ 9'$	$11^0\ 15'$
North Kollam	*murāri* ($2^{a}\ 25^{v}$)	$11^0\ 23'$	- -

The auto-commentary refers to the Tropic of Cancer where the shadow never goes to the South and Arctic Circle at which the day lasts for 60 *nāḍikās* when it is maximum.

AKṢA AND *LAMBA*

28. The square root of the sum of the squares of *śaṅku* and *palabhā* is *śaṅkukarṇa*. Dividing by it 41253, (*guṇaramyabhā*), *lambaka* is obtained. *Akṣajyā* is obtained by multiplying *palabhā* by *trijyā* and dividing by *śaṅkukarṅa*.

$$\textit{śaṅkukarṇa} = \sqrt{144 + 144\tan^2\varphi}$$

$$= 12\sec\varphi$$

$$\textit{lambaka} = \frac{41253}{12\sec\varphi} = \frac{R\times 12}{12\sec\varphi} = R\cos\varphi$$

$$\textit{akṣajyā} = \frac{\textit{palabhā} \times 12}{\textit{śankukarṇa}}$$

$$= \frac{12 \tan \varphi \times R}{12 \sec \varphi}$$

$$= R \sin \varphi$$

COMPUTATION OF *CARA* AND OTHERS

29. Multiply *akṣajyā* by *trijyā* and divide by *lambajyā*. The result is called *svadeśa guṇakāraka*. When the square of *trijyā* is divided by *lambajyā*, *svadeśahāraka* is obtained. Find the *bhujajyā* and *koṭijyā* of the declination of the planet. Then *carajyā* is obtained by multiplying *guṇakāraka* by *bhujajyā* of the declination and dividing by *koṭijyā* of the declination.

$$\textit{svadeśa guṇakāraka} = \frac{\textit{akṣajyā}}{\textit{lambajyā}} \times \textit{trijyā}$$

$$= \frac{R \sin \varphi}{R \cos \varphi} \times R = R \tan \varphi$$

$$\textit{svadeśahāraka} = \frac{R^2}{\textit{lambajyā}} = \frac{R^2}{R\cos} = R\sec$$

$$\textit{carajyā} = \frac{\textit{guṇakāraka} \times R \sin \delta}{R \cos \delta} = R \tan \varphi \times \tan \delta$$

where δ is the declination.

30. When *palabhā* is multiplied by $\frac{1}{12}$ of *krāntijyā, bhūjyā* is obtained. When this is multiplied by *trijyā* and divided by *dyujyā*, *carajyā* is obtained. Its arc is called *caraprāṇa*.

Another method of finding *carajyā* and *cara* is given. By the rule,

$$bh\bar{u}jy\bar{a} = palabh\bar{a} \times \frac{1}{12} \times R\sin\delta$$

$$= 12\tan\varphi \times \frac{1}{12} \times \sin\delta$$

$$= R\tan\varphi\sin\delta$$

$$cara jy\bar{a} = \frac{R\tan\varphi\sin\delta \times R}{dyujy\bar{a}}$$

$$= \frac{R\tan\varphi\sin\delta \times R}{R\cos\delta} = R\tan\varphi\tan\delta$$

Carajyā gives the ascensional difference at the time of rising or setting.

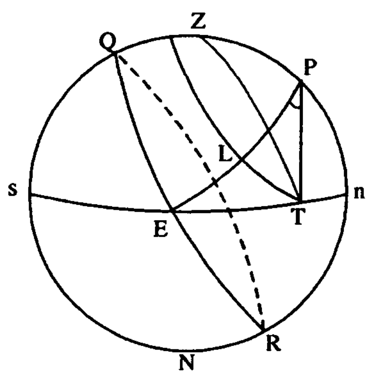


Figure 4.7

Let T be a point at rising in the diurnal path, a small circle parallel to the equator. Let the declination circle PE meet the small circle at L. $\angle LPT$ = *cara*. From Δ PZT,

$$\cos ZT = \cos PZ \cos PT + \sin PZ \sin PT \cos \angle ZPT$$

i.e. $$0 = \cos(90^\circ - \varphi)\cos(90^\circ - \delta) + \sin(90^\circ - \varphi)\sin(90^\circ - \delta)\cos \angle ZPT$$

i.e. $$\cos \angle ZPT = -\tan\varphi \tan\delta$$

i.e. $$\angle LPT = ZPT - 90^\circ = \sin^{-1}(\tan\varphi \tan\delta)$$

$$carajy\bar{a} = R \tan\varphi \tan\delta$$

This is positive if δ is positive and negative if δ is negative. In other words, if the *sāyana* longitude of the Sun is *Meṣādi*, it is negative and if it is *Tulādi* it is positive.

COMPUTATION OF *LAMBAJYĀ*

31. Multiply the *palabhākarṇa* by R^2, divide by *trijyā* and subtract the result from the result divided by *dyujyā*. Divide it by 48. The result is *hārajyā*.

palāṅgulakarṇa $= \sqrt{h^2 + h^2 \tan^2\varphi}$

$= h \sec\varphi$,
where h is the height of *śaṅku*.

hārajyā $$= \frac{1}{48}\left[\frac{R^2 h}{R\cos\delta\cos\varphi} - \frac{R^2 h}{R\cos\varphi}\right]$$

$$= \frac{Rh}{48}\left[\frac{1-\cos\delta}{\cos\delta\cos\varphi}\right]$$

Taking $h = 12$, we get

$$\textit{hārajyā} \quad = \quad \frac{R}{4}\left[\frac{1-\cos\delta}{\cos\delta\cos\varphi}\right].$$

Normally *hārajyā* is defined as $\frac{R(1-\cos\delta)}{\cos\delta\cos\varphi}$ and in the computation of solar eclipses, ¼ of this is required. Hence the definition of the author. However both the methods were in use *vide : Karaṇapaddhati* (VIII 24):

trijyāvargeṇāhatādakṣakarṇād
dyujyābhaktās trijyakābhaktahīnāḥ |
mānyādijyāḥ sambhṛtākṣ'etradeśe
devāptāstā hārajīvā ināḍyāḥ ||

In other words

$$\textit{hārajyā} = \frac{Rh(1-\cos\delta)}{\cos\varphi\cos\delta}.$$

It is also said that *mānyādijyās* are obtained by taking *akṣajyā* as 647. (This is true for Alathur) and *inādijyās* are obtained dividing it by 48. In general, both the definitions are used.

COMPUTATION OF *LAMBANAJYĀ*

32. Multiply the *lambajyā* by 1326 (*candralaya*) and divide by *trijyā*. This is called *caramaphala*. Find the square of this add it to the square of *trijyā*. Subtract from it twice the product of *caramaphala* and the *koṭijyā* (of the arc concerned). Find the root of this and divide by *trijyā* multiplied by *bhujajyā* of the arc. The result is called *grahalambanajyā*.

Let ℓ be the longitude of the planet.

caramaphala $= \dfrac{1326\,.\,h\cos l}{R} = c$ (say)

Find $\sqrt{R^2 + c^2 - 2R\cos \ell.c}$

lambanajyā $= \dfrac{\sqrt{R^2 + c^2 - 2cR\cos \ell}}{R^2 \sin \ell}$

CHĀYATAḤ PŪRVĀPARA REKHĀ

33. Multiply the *bhujajyā* of the *sāyana* Sun with appropriate sign (depending on whether it is *Meṣādi* or *Tulādi*) by the *bhujajyā* of maximum declination and the hypotenuse of the shadow and divide by *lambaka*. Divide the result by *trijyā*. Add to this or subtract from this *palabhā* as the case may be. This is called *bhābhujā*. Draw a circle with the shadow as diameter and mark a distance equal to *bhābhujā* from the tip of the shadow on the circle. Join the point of intersection to the base of the *śaṅku*. It is the East - West line.

It is to be observed that the *palabhā* is to be taken as positive or negative according as the longitude of the *sāyana* Sun is *Tulādi* or *Meṣādi*. The shadow will be towards the north or south according as the longitude of the *sāyana* Sun is *Tulādi* or *Meṣādi*. The

$$bh\bar{a}bhuj\bar{a} = \left| \pm h\tan\varphi + \frac{R\sin \ell R\sin\omega\sqrt{h^2 + s^2}}{R.R\cos\varphi} \right|$$

where ℓ is the *sāyana* Sun, φ is the latitude of the place and ω is the maximum declination, h is the height of the *śaṅku* and s is the length of the shadow.

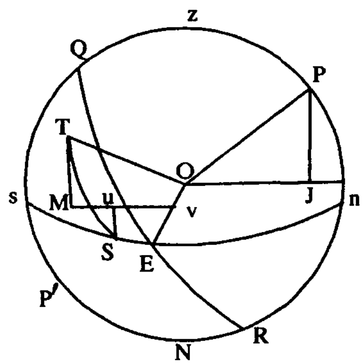


Figure 4.8

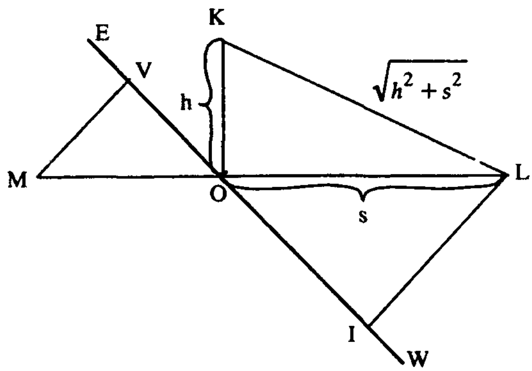


Figure 4.9

Let the Sun rise at *S* and move to *T* in a small circle parallel to the equator during diurnal motion. Draw *TM* perpendicular to the plane of the horizon. Draw a line through *M* parallel to the North-South line. Let the line through *T* parallel to *QR* meet the above line at *U*. Let *OK* be the gnomon and *OL* the shadow. Draw *PJ* perpendicular to the line *OW*. Let *EO* meet *MU* at *V*. For convenience the gnomon and the shadow have been shown in a separate diagram (Fig. 4.9).

We need the value of MV which is the R sine of the angle made by OM with the East-West line. We have from $\Delta\ SsP'$, (P' being the point diametrically opposite to P in Fig. 4.8).

$$\cos P'S = \cos P's.\cos Ss$$

i.e. $$\sin\delta = \cos\varphi\ \cos Ss$$

i.e. $$\cos Ss = \sin\delta\ \sec\ \varphi$$

Consequently,

$$\begin{aligned} UV &= R\sin(90^0 - Ss) \\ &= R\cos Ss \\ &= R\sin\delta\sec\ \varphi \\ &= \frac{R\sin\ell\sin\omega}{\cos\varphi} \quad \ldots \qquad (1) \end{aligned}$$

We shall find MU now.

$$\Delta\ TMU\ |||\ \Delta\ OJP.$$

Therefore,

$$\frac{TM}{OJ} = \frac{MU}{JP}$$

We get,

$$\begin{aligned} MU &= TM\ .\frac{JP}{OJ} \\ &= TM\ \frac{R\sin\varphi}{R\cos\varphi} = TM\tan\varphi \quad \ldots \qquad (2) \end{aligned}$$

Also,

$$\Delta\ TMO\ |||\ \Delta\ KOL$$

Therefore,

$$\frac{TM}{KO} = \frac{OT}{KL}$$

i.e. $$TM = \frac{KO.OT}{KL}$$

$$= \frac{h.R}{\sqrt{h^2 + s^2}} \quad \ldots \quad (3),$$

where $OK = h$, and $OL = s$. Substituting in (2), we get

$$MU = \frac{h\,R \tan \varphi}{\sqrt{h^2 + s^2}} \quad \ldots \quad (4)$$

Let LI be the perpendicular to the East-West line. Let α be the altitude of the Sun. Then

$$LI = OL \sin LOI$$

$$= KL \cos \alpha \sin \angle LOI$$

Also

$$MV = OM \sin \angle LOI$$

since $\angle MOV = \angle LOI$. Therefore

$$\frac{LI}{MV} = \frac{KL \cos\alpha \sin \angle LOI}{OT \cos\alpha \sin \angle LOI}$$

$$= \frac{\sqrt{h^2 + s^2}}{R}$$

Therefore,

$$LI = MV \cdot \frac{\sqrt{h^2 + s^2}}{R} = (MU + UV)\, \frac{\sqrt{h^2 + s^2}}{R}$$

$$= h \tan \varphi + \frac{\sin \ell \sin \omega}{\cos \varphi} \sqrt{h^2 + s^2}$$

$$= h \tan \varphi + \frac{R \sin \ell \, R \sin \omega}{R.R \cos \varphi} \sqrt{h^2 + s^2} \quad \ldots \qquad (5)$$

If the *palabhā* is negative we get

$$MV = - h \tan \varphi + \frac{R \sin \ell \, R \sin \omega}{R.R \cos \varphi} \sqrt{h^2 + s^2}$$

DEŚĀNTARASAMSKĀRA

34. The prime meridian is the line drawn from the earth's equator, northwards and southwards passing through Laṅkā, Rāmeśvaram, Ujjainī and Meru. The *deśāntara karma* is done with respect to this, and it is positive on the west and negative on the east.

The purpose of *deśāntara* correction is to know the difference in the times due to longitudinal difference. If the planetary position at sunrise at Laṅkā is known, the sunrise takes place earlier at a place with eastern longitude and the corresponding quantity has to be subtracted from the longitude at Laṅkā. Thus the correction is negative.

35. The circumference of the parallel of latitude through the place is equal to circumference of the equator which is equal to 3299 (*dhūrdhuraga*) *yojanas* multiplied by *lambajyā* and divided by *trijyā*. Find the distance eastwards or westwards of the place concerned from the prime meridian, multiply by 60 *nāḍikās* and divide by the circumference of the parallel of latitude to get the quantity of *deśāntara samskāra*.

If the eastern longitude is ℓ, then the correction required is $\frac{\ell}{15}$ hours. In the Indian method, the longitudinal difference

from the prime meridian is not expressed as an angle but as the distance in *yojanas*. When this is divided by the circumference of the equator, the angle is obtained.

The circumference of the parallel of latitude

$$= \frac{3299 \times lambajy\bar{a}}{R} = \frac{3299 \times R\cos\varphi}{R}$$

$$= 3299 \times \cos\varphi,$$

as required.

36. By observing the Sun at eclipse, the position of which is calculated with the computation of eclipses (vv. 21-32) and for which the longitudinal correction is effected and by knowing that the time is less, it follows that the place is on the west of prime meridian. If it is more, it follows that the place is on the east.

The position is calculated from the zero position, that is Laṅkā. If the place concerned is on the east, it should happen earlier. But the observation shows that the Laṅkā time calculated is more than the time of observation. This fact is emphasized here.

THE DURATION OF *RĀŚIS* AT A GIVEN PLACE

37. Correct the *sāyana* longitude of the Sun with *cara* and *prāṇakalāntara*. Then, subtract from the longitude corresponding to the end of every *rāśi* that of the previous *rāśi*. Convert it into degrees etc. divide by 6 and get in *nāḍikās*. The conversion also can be done using the fact that 1 *rāśi* = 1800 *asus*. Then the durations of *rāśis* are obtained.

A point of the ecliptic on the horizon is said to rise. More precisely the ascending point is defined as the point of intersection of the ecliptic and the eastern horizon. To find the longitude of this point, the duration of each *rāśi*, i.e. the interval between the rising of the first point and last point of the *rāśi* is found out. The longitude of the first point of *Meṣa* is 0^0 and that of the last point is 30^0. The interval between the risings of these two is called the *rāśipramāṇa* of *Meṣa*. Similarly the interval between the risings of the points of the ecliptic corresponding to 30^0 and 60^0 is the *pramāṇa* for *Vṛṣabha* and so on. The durations of these in Laṅkā are determined first. Since the *rāśi* is measured along the ecliptic and time along the equator, |Right Ascension – longitude| is determined. This is called *Prāṇakalāntara*. This is additive or subtractive according as the longitude is in the even quadrant or odd quadrant.

Thus we get the *rāśi pramāṇas* for Laṅkā. For a desired place of latitude φ , the *cara* has to be added or subtracted as the case may be. sin(*cara*) = tan φ tan δ. This is negative or positive according as the longitude is in $0° - 180^0$ or $180° - 360°$.

Thus we get the *rāśimāna* for each *rāśi*. In the above stanza the method of getting *sāyana rāśimānas* is given. The *nirayana rāśimānas* change continuously because of the precession of the Equinoxes. One can use *sāyana rāśimānas* and later subtract *ayanāmśa* to get the *nirayana* longitude of *lagna*.

DURATIONS OF DAY AND NIGHT

38. Find the *sāyana* longitude of the Sun and find the *cara*. Divide it by 180°. Add it to 30 *nāḍikās* if it is *Meṣādi* and if it is *Tulādi*, subtract from it. The result is the duration of the day. By subtracting it from 60 *nāḍikās*, the duration of night is obtained.

The east hour angle of the Sun, H at the time of rising

$$= \cos^{-1}(-\tan\varphi \tan\delta) = 90^\circ + \sin^{-1}(\tan\varphi \tan\delta)$$

$$= 90^\circ + cara$$

Duration in *nāḍikās*, when *cara* is expressed in minutes.

$$= \frac{2(90^0)}{6} + \frac{2(cara)}{360^\circ}$$

$$= 30\ \textit{nāḍikas} + \frac{cara}{180}$$

Cara is positive, when δ is positive (in the modern sense) i.e. when the longitude lies between 0 and 180°, or *Meṣādi* and negative when it lies between 180° and 360° or *Tulādi. Cara* is generally defined to be negative for *Meṣādi* and positive for *Tulādi.* Here that principle is reversed. To get *kālalagna* from the Right Ascension of the sun, *cara* is subtracted, if it is *Meṣadi* because rising takes place below the 6 O' clock circle and similarly for the other case. The important thing here is that *Meṣādi* or *Tulādi* indicate opposite signs, but the actual sign depends on the context.

COMPUTATION OF THE ASCENDANT

39,40 To the *sāyana* longitude of the Sun corrected with *cara* and *prāṇakalāntara* add six times the *nāḍikās* elapsed, after sunrise. This is called *kālalagna.* Find the *cara* and *prāṇakalāntara* for the *kālalagna* and correct with the opposite signs. For this find the *cara* and *prāṇakalāntara* in the usual way and correct the *kālalagna.* For this find *cara* and *prāṇakalāntara* and correct with the opposite signs. For this, find the

cara and *prāṇakalāntara* and correct the original *kālalagna*. Continue the process till two consecutive corrections for *kālalagna* are identical. This is the *sāyana lagna*. Subtract *ayanāmśa* to get the *nirayana lagna*.

The ascending point or *lagna* is the point of intersection of the eastern horizon and the ecliptic. *Cara* is positive for *Tulādi* and negative for *Meṣādi*. *Prāṇakalāntara* is negative for odd quadrants and positive for even quadrants as pointed out earlier. *Kālalagna* is the Right Ascension of the East point.

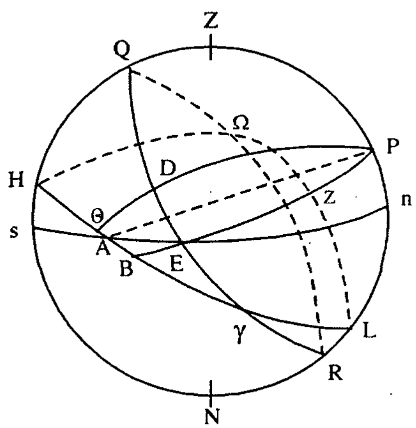


Figure 4.10

In the figure A is the Ascendant. HL is the ecliptic.γ is the first point of Aries. ⊙ is the position of the Sun and D is foot of the declination circle.

Kālalagna γE is measured eastwards. γA measured eastwards is Longitude of *lagna*, γ ⊙ is the longitude of the Sun. When corrected by *prāṇakalāntara* it becomes γD. The Sun rises at a point B on the small circle B⊙ parallel to the equator. The

time after sunrise is multiplied by 6 and added to the arc = $\angle \odot PB$. The *cara* is $\angle BPE$. When this is also added we get γE, the Right Ascension of the East point. To get γA from γE we use the method of successive approximation described earlier.

DIFFERENT *JYĀS*

41. Multiply square of the desired arc in *tatparās* by 1, and divide by square of 90° (expressed in *tatparās* = 19440000). Subtract this from 36, multiply by the square of the desired arc in *tatparās* and divide by square of 19440000, subtract the result from 1604, multiply this result by the square of the desired arc in *tatparās*, divide by the square of 19440000, and subtract this result from 46817, multiply the result by the square of the desired arc in *tatparās* and divide by the square of 19440000, subtract this from 796926, multiply the result by the square of the arc in *tatparās* and divide by the square of 19440000, subtract the result from 6459641, multiply by the cube of the desired arc in *tatparās* and divide by the cube of 1944000. Subtract the result from the arc in *tatparās*. The result is the *bhujajyā* of the arc in *tatparās*.

We shall denote the constants 1, 36, 1604 etc., by k_6, k_5, k_4, k_3, k_2 and k_1

It is necessary to compare the verses of Mādhava quoted in *Yuktibhāṣā* (pp. 91-9):

vidvāmstunnabalaḥ kavīśanicayassarvārtha śīlasthiro
nirviddhaṅganarendrarunnigadite sve su kramāt pañcasu |
ādhastyād guṇitādabhīṣṭadhanuṣaḥ kṛtyā vihṛtyāntima
syāptam śodhyamuparuparyatha ghane naivam
dhanuṣyantataḥ ||

This also prescribes the same method but with different constants. $\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6$, not given in this stanza. The procedure is this:

$$R\sin a = a - \frac{k_1 a^3}{3!R^2} + \frac{k_2 a^5}{5!R^4} - \frac{k_3 a^7}{7!R^6} + \frac{k_4 a^9}{9!R^8} - \frac{k_5 a^{11}}{11!R^{10}} + \frac{k_6 a^{13}}{13!R^{12}}$$

In Mādhava's formula the constants are ℓ_1, ℓ_2, ℓ_3, ℓ_4 and ℓ_5. There is no term corresponding to the last term. We note that the constants given in the v. 41 are :

k_1 =	*parvatāḷi śubhatā*	= 6459641‴
k_2 =	*taraḷatāḷasūḥ*	= 796926‴
k_3 =	*sakṛduktavāk*	= 46817‴
k_4 =	*vainateya*	= 1604‴
k_5 =	*calana*	= 36‴
k_6 =	*kānanam*	= 1‴

All are in *tatparās*. Constants in Mādhava's formula are

ℓ_1 = *nirviddhāṅganarendraruk*
= 2220′39″40‴ = 7994380‴

ℓ_2 = *sarvārthaśīla sthiraḥ*
= 273′ 57″ 47‴ = 986267‴

ℓ_3 = *kavīśanicayaḥ*
= 16′ 6″ 41‴ = 58001‴

ℓ_4 = *tunnabalaḥ*
= 33″6‴ = 1986‴

ℓ_5 = *vidvān*
= 44‴ = 44‴

But the constants here are different. One can compare the two as in the table below:

i	ℓ_i	k_i	$\frac{\ell_i \times 4}{5}$	$\frac{\ell_i \times 9}{11}$	$\frac{\ell_i \times 6459641}{7994380}$
1	7994380	6459641	6395504	6526745	6459641
2	986267	796926	789013	806946	798924
3	58001	46817	46401	47456	46934
4	1986	1604	1589	1625	1608
5	44	36	35	36	36

This shows that instead of using k_i's one can use $K_i \times \frac{7994380}{6459641}$ to get the results. The difference is less than 1′ in each case. The other constants $K_i \times \frac{5}{4}$ and $K_i \times \frac{11}{9}$ will give results slightly higher and lower in values respectively. But it is not clear why the author resorted to such a method. He has not commented on the verse and provided the clue for interpretation.

The correct reading should be *ghanakṛtam* which means cubed. Let there be n terms for $a_1, a_2, \ldots, a_n$ given for subtraction. First subtract a_n from a_{n-1}. We get $a_{n-1} - a_n$. Then subtract this from a_{n-2}. Then we get $a_{n-2} - (a_{n-1} - a_n) = a_{n-2} - a_{n-1} + a_n$. By the next subtraction, it becomes $a_{n-3} - (a_{n-2} - a_{n-1} + a_n) = a_{n-3} - a_{n-2} + a_{n-1} - a_n$. Proceeding thus we get the result $a_1 - a_2 + a_3 + \ldots + (-1)^{n-1} a_n$. This is the principle used.

42. Multiply the square of the desired arc in *tatparās* by 5 and divide by the square of 90^0 expressed in *tatparās* ($= 19440000^2$), subtract from 252, multiply by the square of arc expressed in *tatparās*, divide by 19440000^2, subtract from 9192, multiply by the square of the arc in *tatparās*, divide by 19440000^2, subtract

from 208535, multiply by the square of the arc in *tatparās*, divide by 19440000^2, subtract from 2536695 and multiply by the square of the arc in *tatparās* and divide by 19440000^2. Multiply by the square of the arc in *tatparās* divide by 19440000^2 and subtract from 144337005. The result is *utkramajyā*.

This gives the method of finding *utkramajyā* $= R(1-\cos x)$ for the arc x. As before this is not correct. But we invoke Mādhava's formula in which a similar method is given.

We shall denote the constants in the stanza by m_1, m_2, m_3, m_4, m_5, m_6.

$$R(1-\cos x) = \frac{m_1 a^2}{2!R} - \frac{m_2 a^4}{4!R^3} + \frac{m_3 a^6}{6!R^5} - \frac{m_4 a^8}{8!R^7} + \frac{m_5 a^{10}}{10!R^9} - \frac{m_6 a^{12}}{12!R^{11}}$$

In Mādhava's formula, the constants are

n_1 = *ūnadhanukṛd bhūreva* = 4241′ 9″ 0‴ = 15268140‴

n_2 = *mīnāṅgo narasimhaḥ* = 872′ 3″ 5‴ = 3139385‴

n_3 = *bhadrāṅgabhavyāsanaḥ* = 71′ 43″ 24‴ = 258204‴

n_4 = *sugandhinaganut* = 3′ 9″ 37‴ = 11377‴

n_5 = *strīpiśunaḥ* = 0′ 5″ 12‴ = 312‴

n_6 = *stena* = 0 0 6‴

The constants in the v. 42 are:

m_1	= *mananasadbimboṣthapaḥ*	=	12337005
m_2	= *mugdhākṣītilamātranut*	=	2536695
m_3	= *mārgacodī naraḥ*	=	208635
m_4	= *khaḷakeḷi*	=	9192
m_5	= *phaṇātra*	=	252
m_6	= *muni*	=	5

We can observe the following as before.

i	n_i	m_i	$\frac{n_i \times 5}{6}$	$n_i \times \frac{14334005}{15268140}$
1	15268140	12337005	12723450	14334005
2	3139385	2536695	2616154	2947311
3	258204	208635	215170	242407
4	11377	9192	9480	10680
5	312	252	260	292
6	6	5	5	6

This also indicates that multiplying the constants by $\frac{6}{5}$ or $\frac{15268140}{14334005}$ will give approximate values. But one can try to get a better value of the multiplier. It is also to be doubted whether the readings are correct or have undergone distortion in the process of copying.

But the question again is this: why does he give this method? Does he want to give a riddle to the readers who could cudgel their brains and get the solution, instead of telling them the truth?

This also is not commented by the author and there is no clue to interpretation except for a direct investigation like the above.

43. The *jyā* of a desired arc is called *kramajyā*. When it is squared, subtracted from the square of *trijyā* and the root is extracted, *koṭijyā* is obtained. When *kramajyā* is multiplied by *trijyā* and divided by *koṭijyā*, *spṛgjyā* is obtained. When *koṭijyā* is multiplied by *R* and divided by *bhujajyā*, *kuspṛgjyā* is obtained. When the former is squared and added

to the square of *trijyā* and the square root is found, *chedijyā* is found. When *kusprgjyā* is squared and added to the square of *trijyā* and the root is extracted *kucchedijyā* is obtained.[5]

First R sine is defined and it is asserted that $\sqrt{R^2 - R^2 \sin^2 x} = R\cos x$. *Sprgjyā* is defined as $R \tan x$ and *kusprgjyā* as $R\cot x$. We have the result $\sqrt{R^2 + R^2 \tan^2 x} = R\sec x$ which is called *chedijyā* and $\sqrt{R^2 + R^2 \cot^2 x} = R\operatorname{cosec} x$, which is called *kucchedijyā*. The interesting fact is that R tan, R cot, R sec and R cosec are also defined unlike in other Kerala works.

NOTES

1. C.T. Rajagopal and Vedamurthi Iyer, "On the Hindu Proof of Gregory's series", *Scripta Mathematica* 17, 1951, pp. 65 - 74.

   K. Mukunda Marar and C.T. Rajagopal, "On the Quadrature of the Circle" *Journal of Bombay Branch of Royal Asiatic Society* (NS) 20, 1944, pp. 65 - 82.

   C.T. Rajagopal and M.S. Rangachari, "On an Untapped Source of the Medieval Kerala Mathematics" *Souvenir of the 42nd Annual Conference of Indian Mathematical Society,* Thiruvanathapuram, 1976.

   There are other forms of the series for $\frac{\pi}{4}$. For example, $\frac{\pi}{4} = \frac{3}{4} + \left(\frac{1}{3^2} - \frac{1}{5^2-5} + \frac{1}{7^2-7} - \ldots\right)$

   T.A. Saraswati (*Geometry in Ancient and Medieval India*, Motilal Banarsidass, 1979) observes that this can be obtained by rearranging the terms of series $1 - \frac{1}{3} + \frac{1}{5}$ .... which is not, however absolutely convergent. If the series is not absolutely convergent, rearrangement

of terms need not lead to a series with the same sum. It has to be investigated whether any independent proof existed for this. This is important since it opens up the question of more knowledge of Analysis by the Kerala mathematicians.

2. The reading *samīpatajjyayoḥ* is not satisfactory. One can compound *samīpataḥ* with *jyā*, interpret the term as *samipataḥ sthitayoḥ jyayoḥ* and read it as *samīpatojyayoḥ*. The auto commentary gives as *samīpītajyā.*
3. Uniqueness of the constructed figure is not asserted here.
4. *Cf. Muddle of Ayanāmśa* by S. Madhavan, CBH Publication, 1993.
5. The term '*tadvat vyāsārdhavargāt*' should be interpreted as 'similarly from the square of *trijyā*'. But this is also not quite accurate. The term '*tadvat*' is not accurate because, addition of the squares of *spṛgjyā* and *kuspṛgjyā* to the square of *trijyā* is done and not subtraction from the square of *R*. This is clear from the *sūtra* "*Teṅa tulyam kriyā cedvatiḥ*" (*Aṣṭādyāyī* , V.1.115). Even the use of '*koṭi*' to mean *koṭijyā* is not very correct. Auto commentary on this verse is not available. At times this is loosely done, but in a verse which requires clarity, the usage is not satisfactory. Interpretation is done with the aim of getting specific results. Probably a commentary was written by the author to interpret the terms to mean what he has stated, though not available now.

CHAPTER V

ON THE KNOWLEDGE OF FIVE ELEMENTS

COMPUTATION OF FIVE ELEMENTS

1. Shadow (*chāyā*), eclipse (*grahaṇa*), *cakrārdha*, combustion (*mauḍhya*) and the height of the Moon's horns (*śṛṅgonnati*) are called the five elements. The computation of these is now described.

Cakrārdha refers to *lāta* and *vaidhṛta* described later. Combustion is a term applied to planets, when they come near the Sun apparently and become invisible. The times of heliacal rising and setting are determined. *Śṛṅgonnati* literally means the height of horns (of the Moon). The phases of the Moon are determined.

THE SUN'S SHADOW

FINDING *KĀLALAGNA*

2. The *kālalagna* at sunrise is obtained by correcting the *sāyana* Sun by *prāṇakalāntara* and *cara*. To get the *kālalagna* at sunset, add six *rāśis* and do the corrections. For the noon, add three *rāśis* and do the corrections. For other times, choose the nearest of the three (before the concerned time), do the corrections, find *nāḍikās* elapsed after that, multiply by six and add it to the figure obtained earlier.

There is no need for *cara* correction at noon. It is enough to add three *rāśis* to the *sāyana* longitude of the Sun and correct with *prāṇakalāntara*. *Cara* is positive for *Tulādi* and negative for *Meṣādi*. *Prāṇakalāntara* is positive in even quadrants and negative in odd quadrants.

Kālalagna is the right ascension of the East point. *Prāṇakalāntara* = |longitude − right ascension (R.A.)| and *cara* is the ascensional difference given by sin *cara* = tan φ tan δ, where φ is the latitude of the place and δ is the declination of the Sun. These things were discussed in Chapter IV.

We shall discuss these ideas with the following figure:

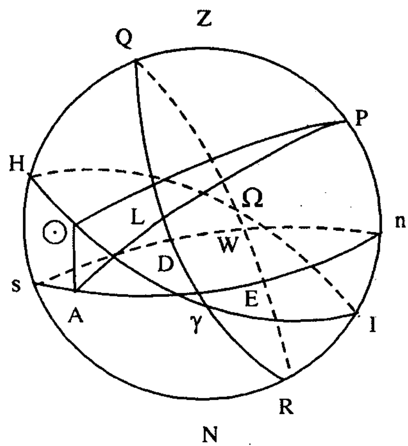


Figure 5.1

In the figure *ns* is the horizon with *n* and *s* as North and South points. *QR* is the celestial equator, *P* being the North Pole. *E* and *W* are the East and West points. *HI* is the ecliptic, γ and Ω are the first point of Aries and the first point Libra respectively. ⊙ represents the Sun on the ecliptic. φ is the latitude of the place and δ is the declination of the Sun.

Let *A* be the point of Sun's rising. Let the great circle *P*⊙ meet *QR* at *L* and let *PA* meet the equator at *D*. The Sun's *sāyana* longitude = γ⊙(eastwards) and R.A. = γ*L* (eastwards).

R.A. of the East point = γ*D* + *DE*

$$DE = 90^0 - \text{East hour angle of } A$$
$$= 90^0 - \cos^{-1}(-\tan\varphi\ \tan\delta)$$
$$= \sin^{-1}(\tan\varphi\ \tan\delta) \text{ numerically}$$
$$= \textit{cara}$$

Therefore,

$$\gamma L = \textit{sāyana} \text{ longitude} \pm \textit{prāṇakalāntara}$$

and

$$\gamma E = \gamma L + LD + DE$$
$$= \gamma L + DE + LD$$

= *sāyana* longitude of the Sun with corrections + 6 multiplied by time elapsed since sunrise.

FINDING THE LARGE SHADOW AND DETERMINATION OF TIME

3 & 4. Find the *sāyana* longitude of the Sun, its *cara* and *apamabāṇa*. When *apamabāṇa* is subtracted from *trijyā*, *dyujyā* is obtained. Subtract the *kālalagna* at the place (with the same longitude) of latitude 0° from the *kālalagna* at the place at the time concerned. Find its *bhujajyā*. Make the correction for *cara*. Multiply this by *dyujyā* and divide by the *hāraka* for the place to get the Sun's *śaṅku*. Square this, subtract from the square of *trijyā* and extract the square root. Multiply this by the height of the *śaṅku* and divide Sun's *śaṅku*. The result is the length of the shadow.

Let δ be the declination of the Sun; then *apamabāṇa* is $R - R\cos\delta$. Then *dyujyā* $= R - (R - R\cos\delta) = R\cos\delta$ and

Sun's *śaṅku* =

$$\frac{\left[R\sin\left[\textit{kālalagna} - \textit{kālalagna at } 0^{\circ} \textit{ at sunrise}\right] \pm \textit{carajyā}\right] R\cos\delta}{R\sec\varphi} = s\,(say)$$

$$\text{Length of the shadow} = \frac{\sqrt{R^2 - s^2}\,.\,h}{s}$$

Let us examine the situation.

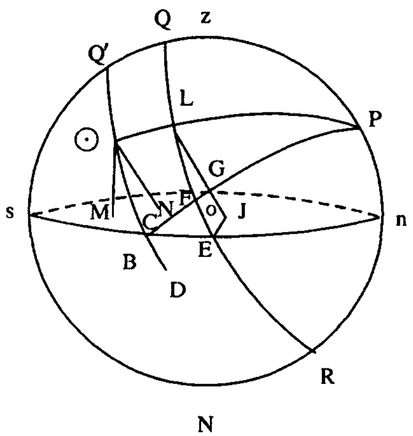


Figure 5.2

O is the position of the observer in Figure 5.2.

The figure is as drawn earlier, but with some additions and deletions. The ecliptic is not shown in the figure. Then ⊙ is on the diurnal path which is a small circle $Q'\odot$ parallel to the equator. Let it meet the horizon at *B* and the declination circle *PE* at *D*. Let *PB* intersect *QR* at *F*. Draw $\odot M$ perpendicular to the plane of the horizon. Draw $\odot N$ in the plane of the small circle $Q'\odot$, perpendicular to *BC*, *C* being the point of intersection of the line *NS* and the plane of the small circle. Draw *EJ* parallel to *BN* such that $\angle EJL = 90^{0}$. Let *G* be on *LJ* such that *FG* is perpendicular to *LJ*.

We observe that $\odot M$ is parallel to ZO and $\odot N$ is parallel to QO. Therefore,

$\angle M\odot N = \angle QOZ = \varphi$. We get

$$\frac{\odot M}{\odot N} = \cos\varphi\ .$$

Also

$$\odot N = LG\cos\delta = (LJ - GJ)\cos\delta$$

$$= (R\sin LE - R\sin FE)\cos\delta$$

$$= [R\sin(\gamma E - \gamma L) - R\sin FE]\cos\delta$$

$$\odot M = \odot N\cos\varphi = \frac{[R\sin(\gamma E - \gamma L) - R\sin FE]R\cos\delta}{R\sec\varphi}$$

In this case the Sun's longitude is *Tulādi* and *cara* is subtractive. If it is *Meṣādi* it will be additive. This is what is done in the Indian method. γE is *kālalagna* and γL is the right ascension of the Sun, if the term *kālalagna* at 0°, at sunrise is interpreted as the *kālalagna* at 0°, at sunrise at the moment concerned. In that case the *kālalagna* concerned is the right ascension of the Sun at rising at that place.

In *Pañcabodha* (VII. 1) :

$$\odot M = \frac{\{R\sin(\text{Time elapsed since sunrise} \times 6) \pm \textit{carajyā}\}R\cos\delta}{R\sec\varphi}$$

The remaining part is simple. Let the Sun's *śaṅku* be s. Let T be the tip of the shadow and OK be the gnomon.

$$\frac{\text{Height of the } \textit{śanku}}{\odot M} = \frac{h}{s}$$

Denoting the length of the shadow by ℓ, we get

$$\frac{\ell}{\sqrt{R^2 - s^2}} = \frac{h}{s}$$ from similar triangles $\odot MT$ and KOT.

and

$$\ell = \frac{\sqrt{R^2 - s^2}\; h}{s}$$

Example

(1) Find the *kālalagna* at Thiruvananthapuram (Lat 8°29′, long 76° 59 E) at 10 *nāḍikās* after the sunrise on 20.4.2004.

Sāyana longitude at sunrise = 30°17′

Declination of the Sun = 11° 34′

$$\textit{prāṇakalāntara} = \frac{R\sin\ell \; R\cos\ell \;\; R(1-\cos\omega)}{R \;\; R\cos\,\delta}$$

Therefore,

$$\sin(\ell - \alpha) = \frac{\sin\ell\,\cos\ell\,(1-\cos\omega)}{\cos\,\delta}$$

$$= \frac{\sin 30°17'\cos 30°17'(1-\cos 24°)}{\cos 11°34'}$$

So, $\ell - \alpha = 2°9'$. This is negative since

$0' < \ell < 90°$

$$\sin\,(\textit{cara}) = \tan\,\varphi\;\tan\,\delta$$

$$= \tan\,8°29'\;\;\tan\,11°34'$$

So *cara* = $1°45'$ and it is negative since $0 < \ell < 180°$. Therefore *kālalagna* = $30° 17' - 1° 45' + 6° \times 10 = 86° 23'$

We shall do this problem using the tables for *prāṇakalāntarajyā* and *carajyā.*

sāyana longitude of the Sun at Sunrise = $30^0 17'$

krāntijyā = $699 + 5 = 704'$

So *krānti* = $11^0 49'$

(The difference is because the obliquity is taken as $24°$ instead of $23° 27'$ in Indian Astronomy)

prāṇakalāntarajyā = $126'$

$= 2° 6'$

This is negative since $30° 14'$ is in the first quadrant.

R sin *cara* = $131' + 3' = 134'$ for Alathur.

For Thiruvananthapuram,

$$R \sin cara = \frac{134}{138} \times 107 = 104'$$

cara = $104' = 1° 44'$

Being *Meṣādi,* it is negative and R sine = arc since the angle is small

kālalagna = $30° 17' - 2° 6' - 1° 44' + 6^0 \times 10$

$= 26^0 27' + 60^0 = 86^0 27'$

(2) Find the length of the shadow.

Sun's *śaṅku* = s

kālalagna – *kālalagna* at 0° at sunrise

$$= 86^{0}\,27' - (30^{\circ}\,17' - 2^{\circ}\,6')$$

$$= 58^{\circ}16'$$

$$= \frac{\left[R\sin(\textit{kālalagna} - \textit{kālalagna}\text{ at }0^{0}\text{ at sunrise}) + \textit{carajyā}\right]\textit{dyujyā}}{\textit{hāraka}}$$

$$= \frac{(2923+104)\times 3369}{3476} = 2932 = s\ .$$

Therefore, $\frac{\sqrt{R^2 - s^2}}{s} = 0.6213$. If the height of the *śaṅku* = 52 then length of the shadow = 0.6213 × 52 = 31.8395 *aṅgulas*.

5. Square the length of the shadow and the *śaṅku*, add them and find the root. Multiply *śaṅku*, *trijyā* and the *hāraka* for the place and divide by *dyujyā*. Note this result. Divide this by the earlier result namely $\sqrt{(\text{shadow})^2 + (\textit{śanku})^2}$. Correct the result with *cara*, find the arc and correct it with *cara* in the opposite sense. Divide by 360. The result gives in *nāḍikās*, time elapsed since sunrise, if it is before the noon and the time before the sunset if after the noon.

$$\sqrt{(\text{shadow})^2 + (\textit{śaṅku})^2} = \sqrt{\frac{(R^2 - s^2)h^2}{s^2} + h^2}$$

$$= \frac{Rh}{s}$$

Also

$$\frac{trijy\bar{a} \times \acute{s}anku \times R\sec\varphi}{R\cos\delta \times \frac{Rh}{s}}$$

$$= \frac{R \times h \times R\sec\varphi}{R\cos\delta \times \frac{Rh}{s}}$$

$$= s\sec\delta\ \sec\varphi$$

From Stanzas 3 and 4, this reduces to R sin [*kālalagna* – *kālalagna* at 0^0 lat. at sunrise + *cara* correction]. Retracing the steps, we get the result.

FINDING *PALĀṄGULA*

6. Find the length of the shadow for the midday, multiply by *trijyā* and divide by the hypotenuse. Find the arc and it is *Meṣādi* if the shadow is in the North and *Tulādi* if the shadow is in the South. Find the declination for the *sāyana* Sun. Add the two if they have the same sign. Otherwise find the difference. The *bhujajyā* of result is *akṣajyā*. When it is subtracted from the square of *trijyā* and the root is extracted, the result is *lambajyā*. $12 \times \frac{ak\d{s}ajy\bar{a}}{lambajy\bar{a}}$ gives the *palāṅgula*.

If the declination of the Sun is δ, the meridian zenith distance is z and the latitude is φ, then $\delta + z = \varphi$, provided δ is considered positive or negative according as it is North or South and the zenith distance is considered positive or negative according as it is South or North. When the shadow is

in the North, the Sun is in the South and conversely. Thus the same convention is followed in the rule given in the stanza.

If s is the length of the midday shadow and t is the hypotenuse, then $R.\frac{s}{t} = R\sin z$ where z is the meridian zenith distance.

The corresponding arc = z. $z + \delta = \varphi$, as noted earlier and

$$R \sin (z + \delta) = R \sin \varphi = akṣajyā.$$

$$lambajyā = \sqrt{R^2 - R^2 \sin^2 \varphi} = R\cos\varphi$$

$$palāngula = 12 \times \frac{akṣajyā}{lambajyā} = 12 \tan \varphi$$

FINDING *ŚAṄKVAGRA* AND *AKXĀGRA*

7. Multiply the Sun's *śaṅku* by *palāṅgula* and divide by 12. This gives the tip of the *śaṅku* (*śaṅkvagra*), south of the line joining the rising and setting points. The *arkāgra* is obtained when the *bhujajyā* of the *sāyana* longitude of the Sun is multiplied by the maximum declination and divided by *lambajyā*.

$$śankvagra = \frac{\text{Sun's } sanku \times palāngula}{12}$$

$$arkāgra = \frac{R\sin \ell \, R\sin \omega}{R\cos\varphi}$$

LONGITUDE OF THE SUN IN PRIME VERTICAL

8. The *śaṅku* of the Sun when meeting the prime vertical is given by

$$\frac{arkāgra \times lambajyā}{akṣajyā} \quad \text{or}$$

$$\frac{krāntijyā \times trijyā}{akṣajyā}$$

Let ℓ be the *sāyana* longitude of the Sun. Then

$$\frac{arkāgra \times lambajyā}{akṣajyā} = \frac{R\sin\omega\sin\ell\, R\cos\varphi}{\cos\varphi\, R\sin\varphi}$$

$$= \frac{R\sin\omega\sin\ell}{\sin\varphi}$$

Also $$\frac{krāntijyā \times trijyā}{akṣajyā} = \frac{R\sin\delta \times R}{R\sin\varphi}$$

$$= \frac{R\sin\omega\sin\ell}{\sin\varphi}$$

This is a result well-known in Astronomy.

9. When *sama śaṅku* is multiplied by *akṣajyā* and divided by R sine of maximum declination, the corresponding arc gives the *sāyana* longitude of the Sun or 360° – the longitude.

$$\frac{samaśaṅku \times akṣajyā}{R\sin\omega} = \frac{R\sin\ell\sin\omega \times R\sin\varphi}{R\sin\omega\ \sin\varphi}$$

$$= R\sin\ell$$

The corresponding arc (180° – arc) or (360° – arc) gives the longitude of the Sun.

FINDING THE LONGITUDE OF THE SUN FROM THE SHADOW

10. Find the length of the shadow of the Sun at noon. Multiply by *trijyā* and divide by the hypotenuse. Find the corresponding arc. It is the *nata* or meridian zenith distance. If the Sun is in the North add *akṣacāpa* (latitude expressed as arc) and subtract if the Sun is in the South. The result is actually the declination. Multiply this by *trijyā* and divide by the *bhujajyā* of maximum declination. Find the arc corresponding to this, which gives the *sāyana* longitude of the Sun, six *raśis* added to it or 360° – the longitude.

If s is the length of the shadow, z the meridian zenith distance and t the hypotenuse, then

$$\sin z = \frac{s}{t}$$

$$\sin^{-1} \frac{s}{t} = z = \delta \sim \varphi ,$$

where δ is the declination and φ is the latitude of the place. Therefore $z \sim \varphi = \delta$. If the Sun transits North of the meridian, φ has to be added, otherwise it has to be subtracted. This can be verified by drawing suitable figures. From the value of δ, the *sāyana* longitude of the Sun is obtained from the formula.

$$\sin \ell = \frac{\sin \delta}{\sin \omega} \quad \ldots \qquad (1)$$

Since δ can be positive or negative, $\ell = 180° +$ longitude given by (1) or $360° -$ the longitude given by (1).

THE MOON'S SHADOW

LATITUDE OF THE MOON

11. Find the *Dṛk* longitudes of the Sun, the Moon, the Moon's *mandocca*, and add *ayanamśa* to these. Also get the *kālalagna*. Subtract the *sāyana* longitude of Rāhu from that of the Moon and multiply its *bhujajyā* by 270 (*āsura*) and divide by *trijyā*. We get the latitude of the Moon.

Rāhu and Ketu are the nodes of the Moon's orbit. If i is the inclination of this to the ecliptic, then its latitude can be obtained thus. Let M be the Moon and D the foot of the secondary to the ecliptic. Let i be the inclination of the Moon's orbit, and N the node (Rāhu). Let m, and n be the *sāyana* longitudes of M and N respectively.

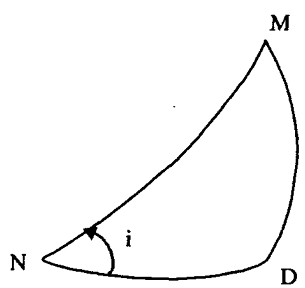


Figure 5.3

Then $\dfrac{\sin MD}{\sin NM} = \sin i$

Therefore sin MD = sin $(m - n)$. sin i, i is taken as 270′ and since it is small we make approximations. We get the latitude

$$MD = 270'.\ \sin(m - n)$$

$$= \frac{270'\ R\sin(m-n)}{R}$$

as given in the stanza. This is positive or negative according as $(m - n)$ is *Tulādi* or *Meṣādi.*

Mandocca is the apogee, the point on the orbit which is farthest from the earth. The *Dṛk* system was introduced by Alathur Paramesvaran Nampootiri, when the earlier system *Parahita* became inaccurate. Since the computation of the shadow has to be done with accurate positions, the author has insisted on the computation by *Dṛk* system.

12. Add three *rāśis* to the *sāyana mandocca* of the Moon. Subtract i. from the longitude of the *sāyaɪ.a* Sun and the longitude of the *sāyana* Moon. Find the *bhujajyā* for the Moon in either case, multiply them and divide by 527 (*sambhrama*). If both the *bhujajyās* have the same direction, add it to the *sāyana* longitude of the Moon. Otherwise subtract from the *sāyana* longitude of the Moon. The result is called the last Moon.

This is also called the second Moon or *dvitīya candra.* The *naronvādijyā* which is used for computing the Moon's true position is used here. It is positive if the arc is *Tulādi* and negative if it is *Meṣādi.*

13. Subtract three *rāśis* from the last Moon. Find the lunar *jyā* and the direction. Multiply by the latitude and divide by 7705. This is called *dṛkphala* and is

positive or negative according as the *jyā* and latitude have the same or different directions.

14. Find half of *dṛkphala* and correct the Moon's longitude obtained earlier. Add or subtract as the case may be. This is the corrected Moon. Find the declination for that longitude. Correct this with the latitude. If both have the same direction, add them. Otherwise subtract the smaller from the larger. From these, *carajyā* and *dyujyā* can be obtained. The moon's position has to be corrected with the whole of *dṛkphala.*

The second Moon and corrected Moon are found out to make the Moon's position accurate.

THE LARGE SHADOW OF THE MOON

15. Find the *prāṇakalāntara* and correct the corrected longitude of the Moon. Then subtract it from *kālalagna.* Find its *mahājyā* and correct it with the *cara* obtained earlier. If both are *Meṣādi* or *Tulādi* add. Otherwise subtract the smaller from the larger. Multiply by *dyujyā* and divide by the *hāraka* for the place. The result is the Moon's *śaṅku.*

SHADOW OF THE MOON AT THE DESIRED TIME

16. If the direction of the *śaṅku* is *Meṣādi* , the Moon is visible. Otherwise it is not visible. Square the Moon's *śaṅku* and subtract from the square of *trijyā* and extract the square root. This is called the large shadow of the Moon. Multiply this by the height of the *śaṅku* and divide by the Moon's *śaṅku* minus four times the motion of the Moon.

Four times the Moon's motion is actually 4 times the motion in a *nāḍikā* or 1/15 of the daily motion. This is clear from *Candrachāyāgaṇita* (VI. 14) of Nīlakaṇṭha Somayājin. This is to account for parallax, though the method is only approximate.

Example

(1) We shall find the length of the shadow in moonlight on 31.5.2004 at Thiruvananthapuram (lat. 8°29′ N, long. 76° 59 E) 12 *nāḍikās* 30 *vināḍikās* after sunset (6-21 PM I.S.T.).

sāyana longitude of the Sun

= *nirayana* longitude + *ayanāmśa*

= 45° 33′ + 23° 55′

= 69° 28′

Mean longitude of the Moon = 208° 48′

sāyana mandocca = 103° 16′

Moon's *manda kendra* = 208° 48′ − 73° 21′ = 129° 27′

True longitude of the Moon

= 208° 48′ + Moon's *mandaphala*

= 208° 48′ − Moon's *mandajyā* (129° 27′)

= 208° 48′ − (234′) from tables.

= 208° 48′ − 3° 54’

= 204° 54′

The *sāyana* longitude of the Moon = 204° 54′

(2) We shall now find the *sāyana* longitude of the second Moon. We have

sāyana Sun – *sāyana mandocca*

$= 69° 28' - 103° 16'$

$= 326° 12'$

sāyana Moon – *sāyana mandocca* $= 204° 54' - 103° 16'$

$= 101° 38'$

Moon's *mandajyā* for $326° 12' = +167'$, since it is *Tulādi.*

Moon's *mandajyā* for $101° 38' = -132'$, since it is *Meṣādi.*

The correction $= -38'$

The correction is negative since one is positive and the other negative.

The second Moon $= 204° 36' - 38' = 203° 58'$

(3) *Dṛk phala*:

To find this we need the latitude of the Moon first.

sāyana Moon $= 204° 54'$

sāyana Rāhu $= 30° 1'$

$$\text{Latitude} = \frac{R \text{ sine } (\textit{sāyana} \text{ moon} - \textit{sāyana} \text{ Rāhu})}{R} \times 270'$$

$$= \frac{R \text{ sine } (174° \ 54')}{R} \times 270' = 25'$$

The latitude is *Meṣādi* (negative)

The second Moon – 3 *rāśis*

$= 204° 8' - 90° = 114° 8'$

Lunar *jyā* = 279′

Being *Meṣādi*, this is negative.

$$Dṛk\ phala = \frac{\text{lunar } jy\bar{a} \times \text{latitude}}{7705} = -\frac{279 \times 30'}{7705} = -1'.08$$

Half the *dṛk phala* = − 1′ (rounded off to minutes)

Correcting the second Moon with this we get *sāyana* longitude of the Moon 204°8′ −1′ = 204° 7′

(4) Declination :

Find the declination as for the Sun for 204° 7′, using the table for *krāntijya*. (See Table 4). From the table, the declination corresponding to 204° 7′ −180° = 24° 7′ is 567′. The corresponding arc = 567′ = 9° 27′. This is positive, being *Tulādi*. The latitude = 25′.

The declination of the Moon = + 9° 27′ −25′ = + 9° 02′

We shall now find *carajyā* and *dyujyā*. *Carajyā* can be obtained from table for Alathur and using the result.

Carajyā for Thiruvananthapuram

$$= \frac{Caraj y\bar{a} \text{ for Alathur} \times pal\bar{a}ngula \text{ for Thiruvananthapuram}}{pal\bar{a}ngula \text{ for Alathur}}$$

$$= \frac{117' \times 104}{138} = 88'$$

This is positive.

dyujyā = *mahājyā* of the *koṭi* of 9° 02′ = 3422′

(5) Length of the shadow

We are now in a position to find the length of shadow caused by the Moon.

The corrected longitude of the Moon	=	$204°8' - 1'$
	=	$204°7'$
prāṇakalāntarajyā	=	$104'$
The arc	=	$104'$ (being small)
	=	$1°44'$

Being in the third quadrant, it is negative. Therefore, R.A. of the Moon = $204°7' - 1°44' = 202°23'$

We shall find the *kālalagna.* Being after sunset adding 6 *rāśis* to the *sāyana* longitude of the Sun we get,

$69°28' + 180° = 249°28'$

Correction for *cara* = *carajyā* ($69°28'$) = $203' = 3°28'$

This is positive being *Tulādi.*

prāṇakalāntarajyā = $101'$

prāṇakalāntara = $101' = 1°41'$,

the angle being small.

This is negative being in the third quadrant.

Therefore the *kālalagna* = $249°28' + 3°28' - 1°41' + 6°12.5' = 326°15'$

kālalagna – R.A. of the Moon = $326°15' - 202°23'$

$= 123°52'$

mahājyā = *jyā* ($123°52'$) = $2855'$

The *cara*, being *Tulādi* we get,

$$mahājyā = 2855'-88' = 2767'$$

$$\text{Moon's } \acute{s}anku = \frac{3422' \times 2767'}{3476} = 2724 = s \text{ (say)}$$

$$\sqrt{R^2 - s^2} = \sqrt{3438^2 - 2724^2} = 2097$$

$$\text{Shadow} = \frac{852 \times 2097}{2724 - 53} = 40.82 \; aṅgulas$$

THE TIME FROM THE SHADOW OF THE MOON

17. The *śaṅku* with which shadow is measured can have a height of 52 *aṅgulas* or 6 feet and a half. After having guessed the approximate time, from the shadow, all the things done earlier have to be performed and the hypotenuse which is the root of the sum of the squares of shadow and *śaṅku* has to be obtained.

The approximate time after sunrise can be guessed using the length of the shadow. For this, *vākyas* are given in the texts (see Table 8). After guessing the time, one has to calculate the longitudes of the Sun, the Moon and Rāhu. When the Moon is waxing, one can get the time of setting after sunset by doubling the number of the *tithi.* When the Moon is waning, it will indicate the time of rising. From the length of the shadow one can guess the *nāḍikās* elapsed after moonrise or the time remaining for the setting of the Moon. With this, one can find the longitudes of the Moon, Sun and Rāhu.

18. Multiply the square of the shadow of the Moon by four times the motion per hour and divide by the

hypotenuse. Add to it the *śaṅku* and *trijyā*. Divide this by the hypotenuse to get the Moon's *śaṅku*. Multiply this by *svadeśahāraka* and divide by the Moon's *dyujyā*. Find the *carajyā* of this and add if it is *Tulādi* and subtract if it is *Meṣādi*, and then find the corresponding arc.

The commentary indicates that *śaṅku* here is the same as the *śaṅku* mentioned earlier, in the computation of the length of the shadow.

19 & 20. After finding the *cara* add it to the Moon's position, if it is before the meridian passage. If it is afterwards, subtract the *cara* from 6 *rāśis* and add it to the corrected Moon. From this subtract the *kālalagna* at setting and divide by 6. This will give the *nāḍikās* etc. at the time concerned. If this agrees with the time guessed, the correct result is obtained. Otherwise find the difference and multiply by the rate of motion, get the new positions, repeat the process till concurrent values are obtained.

By retracing the steps we get the results. We have already guessed the time using the foot-*vākyas* and the *tithi*. If the result obtained agrees with it, accept it. Otherwise, use a method of successive approximation. The auto-commentary says that if the time guessed and that arrived at by computation are different, find the difference and multiply by the rate of motion, arrive at the new time (earlier or later from the guessed time as the case may be). For this time, repeat the computation and continue till concurrent values are obtained. In *Pañcabodha*, the method is to find the difference, divide it by 27 and add to or subtract from the guessed time as required and continue till concurrent

values are obtained. The rate of motion divided by 60 is $\frac{13^\circ 10' 35''}{360^\circ}$. When the difference is multiplied by this and added to or subtracted from the guessed time, and computations are done, the spirit of the method is the same. The method seems only to find a point in between the guessed time and the time computed and continue the process. But '*gatighnād bedāmśādatula hṛtam*' actually means multiply the *nāḍikās* by the rate of motion and divide by 360.

COMPUTATION OF ECLIPSES

NORMAL PROCEDURE IN ECLIPSES

21. If the longitude of the Sun minus that of the Moon reduced to the first quadrant is less than the difference in the daily motions of the Sun and the Moon at the end of a Full Moon or a New Moon, eclipse has to be computed. The solar eclipse occurs during day at New Moon and the lunar eclipse during night at Full Moon.

The major and minor solar ecliptic limits are 18° 31′ and 15° 24′ respectively. When the distance of the Sun from the nearer node is less than 15° 24′ at New Moon solar eclipse will occur and if it is greater than 18° 31′ it will not occur. If it is in between the limits, it may or may not occur. The major and minor lunar ecliptic limits are 12° 5′ and 9°30′ respectively and they are explained similarly. If the distance of the Sun from a node is less than 18° 31′, at a New Moon, one has to check the occurrence of solar eclipse and if it is less than 12° 5′ at a Full Moon, the occurrence of lunar eclipse has to be checked. But the above verse fixes the bounds as the difference in the daily motions of the Sun and the Moon, the mean value of which is about 12° 13′ 27″. Though this is sufficient for lunar eclipses,

what about solar eclipses? Though the theoretical value of major ecliptic limit is 18° 31, which depends on factors like latitude of the place, *Gaṇita Nirṇaya* asserts that solar eclipses need be computed only if the distance of the Sun from the nearer node is less than *palāṅgula* increased by 11. This is the normal rule in India.

22. Find the positions of the Sun, the Moon and Rāhu in *Dṛk*, their daily motions, and use *dṛk ayanāmśa*. Add six *rāśis* to the Sun's longitude for lunar eclipses. Then the longitudes of the Sun and the Moon at the end of Full Moon will be equal.

Emphasis on the *Dṛk* system is laid, because an eclipse is an observable phenomenon and observed positions of planets should tally with the computed positions.

THE DIFFERENCE BETWEEN LUNAR AND SOLAR ECLIPSES

23. Find the *sāyana* longitude of the corrected Moon, subtract three *rāśis* from it. Find *carajyā* and *hārajyā*. Add three *rāśis* to the longitude of the Moon and correct it with *prāṇakalāntara*. Subtract *kālalagna* from it. Find its *mahājyā* and correct it with *carajyā* with suitable sign. Divide this by the sum of the *hārajyā* and one-quarter of *svadeśahāraka*. This gives *lambana* in *nāḍikās*. It can be converted into *nāḍikās* and *vināḍikās*.

In solar eclipse geocentric parallax plays an important role. *Lambana* is the change in the time of the eclipse caused by the parallax in longitude. To discuss the rationale of the method here, it is necessary to explain the concept of parallax.

The effect of parallax

We shall first explain the concept of geocentric parallax and proceed to discuss the methods used in Indian Astronomy.

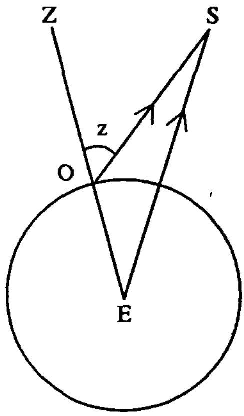


Figure 5.4

Let E be the centre of the earth and a its radius. The observer O on earth observes the body S in direction OS and the observer E at the centre of the earth, in the direction ES. Then the $\angle OSE = p$ is called the geocentric parallax of S. Let $ES = d$. $\angle ZOS = z$ (say). Then we get from ΔOSE

$$\frac{OE}{\sin OSE} = \frac{ES}{\sin EOS}$$

i.e. $$\frac{a}{\sin p} = \frac{d}{\sin(180^\circ - z)} = \frac{d}{\sin z}$$

Thus $\sin p = \frac{a}{d} \sin z = P \sin z$ where $P = \frac{a}{d}$, the horizontal parallax. Clearly, z is the zenith distance of the body. Therefore the effect of parallax is to elevate the body along the vertical circle by an amount $P \sin z$.

Let S be a celestial body which moves to S' due to parallax.

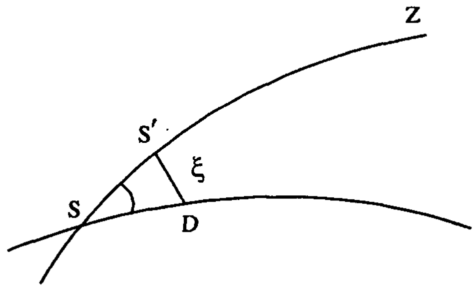


Figure 5.5

Let ξ be angle made by the ecliptic with the vertical. Draw $S'D$ perpendicular to the ecliptic. Treating $SS'D$ as a plane triangle since SS' is small, the resolved parts of the parallax along the ecliptic and the secondary to it are

$$P \sin z \cos \xi \text{ and } P \sin z \sin \xi$$

This shows that the change in the values of the longitude and latitude depend on ξ.

The Indian concept is also the same (*nati lambanayor vāsana*). *Gaṇitayuktayaḥ* (XXI. vv. 2-3) :

dṛṅmaṇḍala kṣepavṛtta bhakūṭākhyāni yāni tu |
teṣām yogagatam kheṭam paśyatyavani madhyagaḥ ||

bhūpṛṣṭhagaḥ punaḥ proktavṛtta tritayayogataḥ |
dṛṅmaṇḍalānusāreṇa lambitam manyate graham ||

This means that the planet in vertical circle or *bhakūṭa* (apexes of circles) cutting the ecliptic at right angles as observed by a terrestrial observer appears deflected from its position as observed by one at the centre of the earth, according to its position in vertical.

In Indian Astronomy, parallax is used only for finding the difference of time caused by it. If *Ś* is towards zenith, the corresponding time will be later to the observed time. Thus it can be treated as deflected away from the zenith. We refer the reader to the demonstration given by T.S. Kuppanna Sastri in his translation of *Pañcaśiddhāntikā* (p. 174).

T.S. Kuppanna Sastri treats it this way, though we have used the modern concept. There is no difference except for the direction.

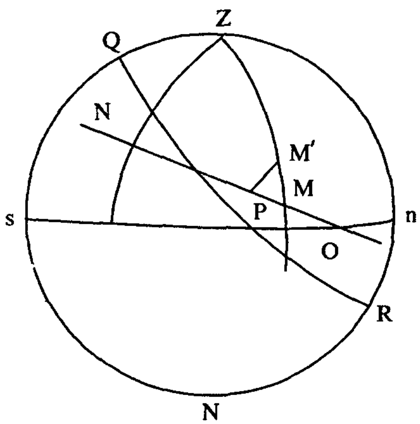


Figure 5.6

Z – Zenith

N – Nanogesimal – *lagna* –90°

O – Orient ecliptic point (*lagna*)

ZM – Zenith distance of M

PM′ – Parallax in latitude

MP – Parallax in longitude

Parallax in latitude

$$PM' = MM' \sin M'MP$$
$$= P \sin ZM \,.\, \sin ZMN$$
$$= P \sin ZN = P \sin Zd\,N \; (N \text{ is } 90^0 \text{ behind } 0)$$

(Zd stands for zenith distance)

Similarly

Parallax in longitude

$$MP = MM' \cos M'MP$$
$$= P \sin ZM \cos ZMN$$
$$= P \,.\, \cos ZN \,.\, \sin MN$$
$$= P \,.\, \cos Zd\,N \,.\, \cos OM \; (OM \text{ being } \textit{lagna} - 90^\circ)$$

N is defined as Nanogesimal, i.e. point 90^0 behind *lagna*. Then ZN is the vertical circle. ZN is secondary to the horizon. ZN is also secondary to the ecliptic. It follows from the fact that the points of the intersection of two circles are poles of the great circle joining their poles. Since parallax can cause delay of 4 *nāḍikās* while on the horizon, it cannot be ignored. But the method of successive approximation introduced to stabilize the value of *lambana* to some extent takes care of the problem.

Parallax is maximum at the horizon. In Indian Astronomy it is assumed that parallax changes uniformly with time. It is maximum at the horizon and it vanishes at the meridian.

We shall discuss the rationale of the computation of *lambana*. *Lambana* measures the difference in the times of observing the celestial object (the Sun or the Moon here) due to parallax in longitude. It is assumed that the change in parallax, as the zenith distance changes from 0^0 to 90^0 is four *nādikās*. By invoking the rule of three, parallax is determined in other cases.

Gaṇita Yuktibhāṣā (p. 204) gives this as the principle. The principle applied is only approximate. The method given in the text is explained now. This is perhaps a method based on the theory discussed above, but with approximations and modifications.

We get

$$= \frac{R \sin \text{[R.A of the sun} + 90^\circ - \text{R.A of the East Point]} - R \tan\varphi \ \tan\delta}{\frac{1}{4}\frac{R}{\cos\varphi} + \frac{1}{4}\frac{R(1-\cos\delta)}{\cos\varphi \ \cos\delta}}$$

$$= 4[\sin\{\text{R.A of sun} + 90^\circ - \text{R.A of the East point}\} - \tan\varphi \tan\delta] \cos\varphi \cos\delta$$

where δ is the declination of the Sun with longitude reduced by 90°.

There are many methods for the computation of parallax and they are not discussed here. (See *Sūryasiddhānta* (5.12) and the discussion thereof).

24. Correct the time of conjunction with *lambana*. Find the *sāyana* Moon etc. for this time and again calculate the *lambana* till concurrent figures are obtained. This is the case with solar eclipses. Since *lambana* and *nati* do not affect the lunar eclipses, the instant of Full Moon (the end of *Pūrṇimā*) corresponds to the middle of the eclipse.

The method of computing *lambana* has been discussed in the last stanza. After getting this, correct the instant of New Moon with this. It is negative before the meridian passage of the Sun and positive afterwards. But the *lambana* has to be stabilized by repeating the process till two consecutive values are nearly equal or equal with respect to the degree of approximation.

COMPUTATION OF THE ANGULAR DIAMETER OF THE DISCS

25. To get the angular diameter of the Sun's disc, multiply the daily motion of the Sun in minutes by 140 and divide by 251. This will be in minutes. For getting the angular diameter of the Moon's disc, multiply the daily motion of the Moon in minutes by 10 and divide by 251. This will be in minutes. Multiply the daily motion of the Moon by 15 and subtract the daily motion of the Sun multiplied by 48. Divide the result by 113. The angular diameter of the section of the shadow cone is obtained.

The angular diameter of a distant body is inversely proportional to its distance. But it has been taken to be directly proportional to the rate of motion. As the body comes nearer, the angular diameter increases and the rate of motion also increases. Therefore the principle appears to be correct in spirit. Let us examine the actual situation. In epicycle theory, the planet moves in a circle, round the earth, which is away from the geometrical centre, by a distance equal to the radius of the *mandavṛtta* or epicycle. This is also equivalent to the motion of the body in a circle called *mandavṛtta*, the centre of which moves in a circle called *kakṣyāvṛtta*.

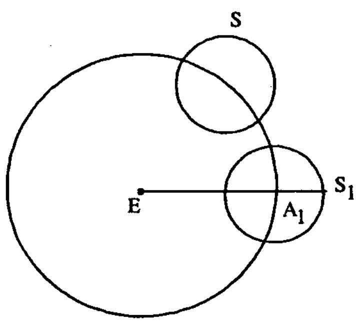


Figure 5.7(1)

In fig 5.7 (1) S_1 is the position at *mandocca*. It takes the same time to trace the *mandavṛtta*, as the centre of *mandavṛtta* to trace the *kakṣyāvṛtta*, but the planet moves in opposite direction in the *mandavṛtta*. The mean planet moves in the *kakṣyāvṛtta* with uniform rate, and the true planet moves in the *mandavṛtta* with the same rate. The equivalent situation is shown in fig 5.7 (2).

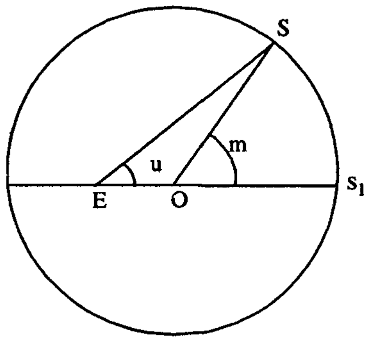


Figure 5.7 (2)

The planet moves in a circle with centre *O*, while the earth is at *E* such that *OE* is the radius of the *mandavṛtta*. S_1 is position at *mandocca*. While the planet moves uniformly in the circle with centre *O* the observer at *E* sees it in the direction *ES* and the motion is not uniform, with respect to *E*. Let $\angle S_1OS$ be m and $\angle S_1ES$ be u. The former is called mean anomaly and the latter true anomaly and $|u - m|$ is called the *mandaphala*. It is observed by Bhaskara II in *Siddhāntaśiromaṇi* (Chedyakādhikāra, v.7) thus :

bhūmermadhye khalu bhavalayasyāpi madhyam yataḥ syād
yasmin vṛtte bhramati khacaro nāsya madhyam kumadhye |
bhūstho draṣṭa na hi bhavalaye madhyatulyam prapaśyet
tasmāt tajñaiḥ kriyata iha taddoḥphalam madhyakheṭe ||

This can be construed thus:

"The centre of the earth which is also the centre of the celestial sphere is not the centre of the circle in which the planet moves. Therefore, the terrestrial observer cannot observe the planet with its mean position. Therefore, enlightened persons have introduced the *doḥphala* to (correct) the mean position."

This is the case with the Sun and the Moon. In the case of star planets this is inadequate. Therefore a *śīghravṛtta* is also introduced.

We shall consider the case of the Sun and the Moon once again. Take ES_1, as the x-axis and y-axis perpendicular to it through E and taking $OE = a$, and radius $= b$ we get the equation of the circle as

$$(x-a)^2 + y^2 = b^2 \qquad \ldots (1)$$

If $ES = r$, we get

$$b^2 = r^2 + a^2 - 2\,ar\cos u \qquad \ldots (2)$$

Differentiating with respect to time t,

$$0 = 2r.\frac{dr}{dt} - 2a\left[-r\sin u.\frac{du}{dt} + \frac{dr}{dt}\cos u\right]$$

i.e.,
$$\frac{dr}{dt} = \frac{ar\sin u}{r - a\cos u} \times \frac{du}{dt}$$

Clearly

$$\left|\frac{ar\sin u}{r - a\cos u}\right| \leq \frac{ar}{r-a} = a\left[1 + \frac{a}{r-a}\right] \qquad \ldots (3)$$

Also, by resolution of velocities along the radius vector and perpendicular to it,

$$\text{the resultant velocity} = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{du}{dt}\right)^2}$$

Since *a* is very small when compared with *r*, so is $\frac{dr}{dt}$ when compared with $r.\frac{du}{dt}$. Thus $\frac{dr}{dt}$ is small and the resultant velocity is nearly $r.\frac{du}{dt}$.

In central orbits, $r^2\frac{du}{dt}$ is constant. But the situation here is quite different. There is a centripetal force towards *O* which is constant. Since the directions of *ES* and *OS* differ by a small angle, we can take them to be the same as an approximation and take $r^2\frac{du}{dt} = k$, a constant. Then $r^2\frac{du}{dt} = k$ and we can assume that angular diameter is proportional to the velocity of the planet (Sun or the Moon). Though *k* depends on the planet, the distance of a planet is defined in the Indian system so as to make *k* unique. (see VI.27).

At *mandocca,* the Sun is farthest from the earth and taking the rate of motion to be 57′ per day, we get by the formula given in the stanza that the angular diameter of the Sun = $\frac{57\times140}{251} = 31'48''$ nearly. Probably this value was obtained from observation and the constants 140 and 251 were derived from that. The modern figure for the same is 31′ 36″ and the difference of 16″ is insignificant in the absence of sophisticated instruments.

The method of finding the angular diameter of a body is given in *Karaṇapaddhati* (VIII. 31) thus:

bimbādīnam yojanāni hatāni tribhajīvayā |
sphuṭayojanakarṇena bhaktānyeṣam kalāḥ smṛtāḥ ||

According to this, the diameter in *yojanas* is to be multiplied by *trijyā* and divided by *sphuṭayojana karṇa* to get the angular diameter.

In particular, the angular diameter of the Sun and the Moon can be obtained by multiplying the daily motion by the diameter and dividing by the daily motion in *yojanas* as evidenced by the following in *Karaṇapaddhati* (VIII. 32) :

athavā sphuṭagatiliptā bimbavyāsasya yojanairguṇitāḥ |
dinayojanagativihṛtās tasya ca liptā bhavanti raviśaśinoḥ ||

The diameter of the Moon is 315 *yojanas* that of the Sun is 4410 *yojanas,* and the daily motion in *yojanas* is 7906.

Angular diameter of the Sun or the Moon

$$= \frac{\text{daily motion} \times \text{diameter in } yojanas}{7906}$$

For the Moon,

$$\frac{\text{diameter in } yojanas}{\text{daily motion in } yojanas} = \frac{315}{7906} \approx \frac{10}{251}$$

For the Sun,

$$\frac{\text{diameter in } yojanas}{\text{daily motion in } yojanas} = \frac{4410}{7906} \approx \frac{140}{251}$$

Thus the above constants are used in computation.

It is necessary to explain the term 'daily motion in *yojanas*'. In Indian Astronomy, there is a principle that every planet moves a distance of 7906 *yojanas* in its orbit daily. The

ākāśa kakṣyā is defined first. It is the circumference of the Universe. *Bhūdina* is the number of civil days in a *caturyuga.* It is defined that circumference of the orbit of a planet

$$\textit{graha kakṣyā} = \frac{\textit{ākaśa kakṣyā}}{\text{no. of revolutions in a } \textit{caturyuga}}$$

$$= \frac{\textit{ākasa kakṣyā}}{\textit{bhūdina}} = 7906 \ \textit{yojanas}$$

See Chapter VI for details.

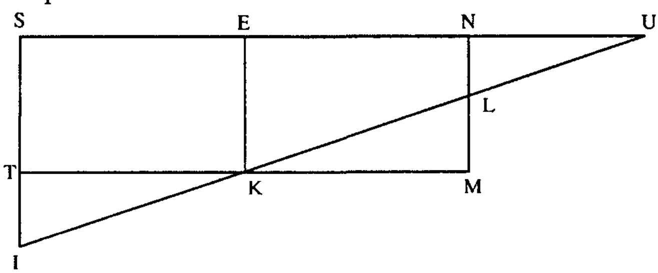


Figure 5.8

Let S be the centre Sun, E the centre earth, N, the centre of section of the shadow, and U the vertex of the cone. The generating line of the cone displayed meets the Sun at I, earth at K, the section of the shadow at L. Draw TKM parallel to $SENU$ to meet SI at T and NL at M. We have

$$\Delta\, TKI \ ||| \ \Delta\, KLM$$

Therefore

$$\frac{TI}{LM} = \frac{TK}{KM}$$

i.e., $$\frac{SI - ST}{NM - NL} = \frac{TK}{KM} \text{ ; i.e. } \frac{2SI - 2ST}{2NM - 2NL} = \frac{TK}{KM}$$

If a, b, c are the diameters of the Sun, Earth and the section of the shadow then we get

$$\frac{a-b}{b-c} = \frac{\text{Distance between the Earth and the Sun}}{\text{Distance between the Moon and the Earth}}$$

We consider the section of shadow along the Moon's path and therefore

$$\frac{a-b}{b-c} = \frac{1/s}{1/m},$$

where s and m are the daily motions of the Sun and the Moon. We get

$$b-c=(a-b)\frac{s}{m}$$

This is the diameter of the section of the shadow in units of length. As in the case of the Sun and the Moon, the angular diameter of the shadow,

$$= c \times \frac{m}{\text{the daily motion in } yojanas}$$

$$= \frac{b \times m - (a-b)s}{\text{daily motion in } yojanas}$$

$$= \frac{1050 \times m - 3360\ s}{7906}$$

(taking $a = 4410$, $b = 1050$ and earth's motion = 7906)

$$\cong \frac{15 \times m - 48\ s}{113}$$, which is the formula given in the text.

COMPUTATION OF *NATI* IN SOLAR ECLIPSES

26. Multiply half the *palāṅgula* by 1287. This is called *ākṣa*. This is always *Tulādi*. Multiply *svadeśahāraka* by 50 and divide by 81. Convert into seconds and divide by the difference in the daily motions of the Sun and the Moon. This is called *natihāraka*.

Find the *kālalagna*, subtract three *rāśis* from it and get its *mahājyā*. If it is *Tulādi* add it to *ākṣa*, otherwise find the difference.

Divide the result by *natihāraka* and using this result correct the latitude according as *nati* is *Meṣādi* or *Tulādi*. This is the corrected latitude of the Moon.

The term used in the stanza is '*pitṛyarkṣakālabhaguṇena*' and this means 'by the *mahājyā* of the *kālalagna* at New Moon' (*pitṛyarkṣa* is New Moon because the manes or *Pitṛs* are supposed to rule over the New Moon). There is no indication for subtracting three *rāśis*. One method to tide over this difficulty is to remove '*digākṣam*' and replace by *tribhona*. The auto-commentary refers to subtraction of three *rāśis*, and so this seems appropriate.

We shall examine the computational procedure. If h is height of the *śaṅku* and φ is the latitude,

$$\text{ākṣa} = \frac{1287}{2} . h \tan\varphi$$

$$= \frac{1287}{2} \times 12 \times \tan\varphi \quad (\text{if } h = 12)$$

$$= 7722 \tan \varphi .$$

One can check that.

$$\frac{3438}{\tan 24^{\circ}} = 7729$$

Thus *ākṣa* $= \dfrac{R \tan \varphi}{\tan \omega}$, allowing for the small difference.

Also

$$\frac{50\ R}{\left(12^{\circ}\ \frac{1}{5}\right).81 \cos\varphi} = \frac{R}{48' \cos\varphi\ \cos\omega}$$

If 48′ is the horizontal parallax the result turns out to be

$$\frac{R}{\text{hor. parallax} \times\ \cos\varphi \cos\omega}$$

This is the rule given in *Pañcabodha* also and is only approximate. Therefore

$$nati\ = \frac{\bar{a}k\d{s}a \pm \mathrm{R} \sin\ (kalalagna - 90^{\circ})}{\dfrac{\mathrm{R}}{\text{hor. parallax } \times \cos\varphi\ \sin\omega}}$$

$$= \text{Hor.parallax}\left[\frac{\tan\varphi}{\tan\omega} \pm \mathrm{R} \sin\ (k\bar{a}lalagna - 90^{\circ}) \cos\varphi \sin\omega\right]$$

$$= \text{Hor.parallax}\left[\sin\varphi \cos\omega \pm \cos\varphi \sin\ (k\bar{a}lalagna - 90^{\circ})\right]$$

THE LATITUDE OF THE MOON AND DURATION OF ECLIPSE

27. Subtract the *sāyana* longitude of Rāhu from that of the Moon, multiply by 4 and find the *jyā*. Divide this by 51. The result is *vikṣepa* for the lunar eclipse. For the solar eclipse find the *nati* and add to the earlier figure or find the difference between the two

according as they have the same sign or not. If the *vikṣepa* is greater than half the sum of the angular diameters, there will be no eclipse. If the figure is less than half the difference of the angular diameters, the eclipse will be full or annular.

The latitude β is given by

$$R\sin\beta = \frac{R\sin(\text{long. of Moon} - \text{long. of Rāhu}) . 270^\circ}{R}$$

At an eclipse, long. of Moon – long. of Rāhu is small or near 180^0. In either case, R sine ≅ difference in longitude in minutes. Thus

$$R\sin\beta = \frac{270}{3438'}\left[R\sin(\text{long. of Moon} - \text{long. of Rāhu})\right]$$

$$\cong \frac{4}{51} R\sin(\text{long. of Moon} - \text{long. of Rāhu})$$

$$= \frac{R\sin(4(\text{long. of Moon} - \text{long. of Rāhu}))}{51}$$

since the difference is small. Computation of *nati* is explained in v. 26.

MEASURE OF ECLIPSE AT A DESIRED TIME

28. Find the difference between the time concerned (corrected with *lambana* or parallax in the case of a solar eclipse) and the accurate time of the end of the syzygy (New/Full moon), convert into *vināḍikās* and multiply by the rate of relative motion of the Moon with respect to that of the Sun. Convert it into

minutes. Find the square of this and add to the square of latitude and find the square root. Subtract the figure (in minutes) from half the sum of the *samparkārdha*. The result is the amount of eclipse at the desired time. In the case of solar eclipse, the eclipse occurs in the direction of *nati* (parallax in longitude) and in the case of lunar eclipse, the eclipse occurs in the opposite direction.

See figure 5.9 given below for the description of lunar eclipse. Consider the position M_2 for example. The distance BM_2 is estimated assuming the motion to be perpendicular to the radius at the middle of the eclipse. This is only approximate.

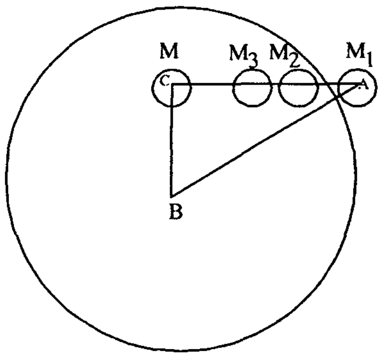


Figure 5.9

HALF THE DURATION OF THE ECLIPSE

29. Find the difference of the squares of the latitude and half of the *samparka*, and extract the square root. Divide this by the rate of the relative motion of the Moon with respect to that of the Sun. The result is *sthityardha* or half the duration of the eclipse. Subtracting this from the time of syzygy and adding

this to that respectively, the times of first point of contact and last point of contact are obtained. The time of syzygy has to be corrected successively by the *lambana*, and the process has to be continued till concurrent figures are obtained.

30. Thus the times of first point of contact and the last point of contact are obtained. In this way, the commencement of total eclipse, or *nimīlana*, the end of total eclipse or *unmīlana* and *vimarda* the duration of total eclipse can be determined. When the angular diameter of the Sun is greater than that of the Moon, the times can be calculated the same way and also the time for filling the periphery of the Sun (in the case of annular eclipse).

We shall summarize the above details now. The theory of eclipses was known to Indians from the Vedic days. Varāhamihira observes in *Bṛhatsamhitā* (v. 8) thus:

bhūcchāyām svagrahaṇe bhāskaramarkagrahe praviśatīnduḥ |
pragrahaṇamataḥ paścānnendorbhānośca pūrvārddhāt. ||

This means that the Moon enters the earth's shadow during a lunar eclipse and the Sun's region during a solar eclipse. That is why a lunar eclipse does not start at the West and a solar eclipse does not start at the East.

In fact, *Tāṇḍya Brāhmaṇa* gives the account of a solar eclipse which when properly interpreted gives the right theory[1].

Lunar Eclipse

A lunar eclipse occurs when the Moon enters the shadow cone of the earth caused by the Sun. To compute the eclipse, we

need the angular diameter of the Moon, that of the section of the cone which the Moon passes through and the latitude of the Moon. These in turn depend on the longitudes of the Sun, the Moon and Rāhu. For the lunar eclipse, the Moon is called *chādya* (*grāhya*) and the earth's shadow is called *chādaka* (*grāhaka*). Half the sum of the diameters of the Moon and the section of the earth's shadow is called *samparkārdha*. The features of the lunar eclipse remain the same throughout the world and it can be given in terms of a universal time. But in ancient India, time was measured by the *nāḍikās* and *vināḍikās* elapsed after sunrise or sunset, at the place and it had to be calculated for the place concerned. One can describe it in terms of I.S.T. now. But, the *nāḍikās* and *vināḍikās* after sunrise and sunset would vary from place to place.

The procedure for the computation of the lunar eclipse is as follows. A lunar eclipse occurs at a Full Moon. So the Full Moon or the exact opposition, which is the time of the middle of the eclipse, should occur at night for the middle of the eclipse to be observable. Find the exact time of opposition, when the Sun and the Moon differ by 180^0. Find the *sāyana* longitudes of the Sun, the Moon, Moon's *mandocca* and Rāhu. For Rāhu, only the mean position is used in Indian Astronomy. As already pointed out, the *deśāntara*, *cara* and *bhujāntara* corrections are required if the intention is to calculate the time according to traditional methods, in which the planetary positions are calculated for Laṅkā, the zero position and reduced to other places. In modern days, we can use the standard time for all these positions. In modern days geocentric longitudes are used and the parallax has to be used to know the nature of the eclipse. But parallax is not significant in a lunar eclipse. First of all, add six *rāśis* to the longitude of the Moon. Then it will be equal to that of the Sun. This is for simplifying computations. The result is called *sāyana*

samakalā. Find the daily rates of motion of the Sun and the Moon and the difference.

We find the angular diameters of the Moon and the section of the shadow. We also find the *samparkārdha*. Find the latitude of the Moon, at the instant of *sāyana samakalā* or opposition. This corresponds to the middle of the eclipse. If the latitude of the Moon at the instant is more than the *samparkārdha*, the eclipse does not occur. When the latitude is subtracted from *samparkārdha*, we get the maximum obscuration (*paramagrāsa*). By dividing this by the Moon's angular diameter, the *pramāṇa* (magnitude) of the eclipse is obtained. Find half the difference of the angular diameters of the shadow and the Moon. It is called *bimbāntarārdha*. If the latitude of the Moon is less than this, the eclipse will be total. We have now got the time of the middle of the eclipse. Now find *sthityardha* or half the duration of the eclipse. For that, square the latitude of the Moon, subtract from the square of *samparkārdha* and divide by the difference in the daily motion of the Sun and the Moon. By subtracting this from the time of the middle of the eclipse, we get time of commencement called *sparśa* (the first point of contact). By adding this to the time of the middle of the eclipse, we get the end of the eclipse called *mokṣa* (the last point of contact). To get the total duration of the eclipse called *vimarda*, square latitude, subtract from the square of *bimbāntarārdha* and find the root. Divide by the rate of difference of motion of the Sun and Moon. This gives half the duration of the total eclipse (*vimardārdha*). By subtracting from and adding to the time of the middle of the eclipse we get the beginning and the end of the total eclipse, which are respectively called *unmīlana* and *nimīlana*. These figures are only approximate. The reason is that the longitude, the latitude, the daily rate of motion, and the angular diameters of *chādya* and *chādaka* change during the eclipse. To get accurate values, we use the method of successive

approximation. We have already got the time of commencement. This is only approximate. Find now the latitude of the Moon at this time, square it, subtract from *samparkārdha* and find the root and divide by difference in the daily motions of the Sun and the Moon, and subtract this quantity from the time of the middle of the eclipse. If it is same as the earlier one accept it. Otherwise, continue till concurrent answers are obtained. Do it for the end, and commencement and end of totality (i.e., when the whole disc is obscure) also.

The figure 5.9 gives the section of the shadow. M_1 is the position of the Moon at the commencement of the eclipse; with A as centre. $M_2 M_3$ and M are positions of the Moon imagined to move in a straight line till it reaches the middle of the eclipse. Let C be the centre of M and B that of the circular section of the shadow. $\angle ACB$ is taken to be 90^0. Then

$$\textit{śthityardha} = \frac{AC}{\text{Difference in the rates of motion}}$$

$$= \frac{\sqrt{AB^2 - BC^2}}{\text{Difference in the rates of motion}}$$

Since $AB =$ *samparkārdha* and BC is the latitude, we get the result used in the method.

Solar eclipse

In the case of solar eclipses, the procedure is more complex. As in the case of the lunar eclipse, find the *sāyana* longitudes of the Sun, the Moon and Rāhu. Find the exact time of conjunction. There is no need to add six *rāśis* to the longitude of the Moon. Find the angular diameters of the Sun and the Moon. In the case of a solar eclipse, the Sun is the *chādya* (*grāhya*) and the Moon is the *chādaka* (*grāhaka*). The sum of the angular

diameters of the Sun and the Moon is the *samparkārdha* here. But, the effect of parallax has to be found out. For this proceed as follows.

Find the *palāṅgula* for the place, and convert into *vyaṅgulas* (1 *aṅgula* = 60 *vyaṅgulas*). Multiply half the *palāṅgula* by 1287. This is called *ākṣa* and is actually equal to $\frac{R.\tan\varphi}{\tan\omega}$, where ω is the obliquity. This can actually be verified by taking the height of gnomon to 12 *aṅgulas*. Multiply the *svadeśahāraka* = $R\sec\varphi$ by 50 and divide by 81. Convert into seconds and divide by the difference in the daily motions of the Sun and the Moon. This is called *natihāraka* or *bhedacit*. This is equal to

$$\frac{R}{\text{hor. parallax} \times \cos\varphi \times \sin\omega}.$$

Then find *cara,* and *prāṇakalāntara* for the longitude of the Sun (and of the Moon). Also multiply by six the *nāḍikās* elapsed after sunrise (at the time of the New Moon) and add it to the earlier figure to get *kālalagna*. Subtract three *rāśis* from the *sāyana* Moon, find *carajyā* and *hārajyā*, equal to $\frac{R(1-\cos\delta)}{\cos\varphi\ \cos\delta}$. Add ¼ of the *svadeśahāraka* and add it to *hārajyā*. There is no need to know the direction of *hārajyā*. It is always *Tulādi*. Find the *cara* and *lambana hāraka* with appropriate signs.

$$\textit{Lambana hāraka} = \frac{R}{\text{hor. parallax} \times \cos\varphi\ \cos\Delta}$$

where Δ is the declination of the Sun corresponding to the longitude-90^0. Add three *rāśis* to the *sāyana* Moon, find *prāṇakalantara* and correct it. Subtract *kālalagna* from that and find *mahājyā*. Find *lambana* as detailed below:

Correct the earlier *mahājyā* with *carajyā* obtained earlier. Subtract three *rāśis* from the Moon's longitude and note the figure (x). Add three *rāśis* to the Moon's longitude, correct it with *prāṇakalāntara* and subtract the *kālalagna* and note the figure (y). If the two figures (x and y) have the same direction, add them. Otherwise take the difference. Note the sign. Divide it by *lambanahāraka*. It gives the *nāḍikās* etc., indicating *lambana*. This has the same direction as the *mahājyā* corrected with *cara*. *Lambana* before the meridian passage of the Sun is negative and that after meridian passage of the Sun is positive and this is the first *lambana* and this needs the correction by iteration. For this, multiply the *lambananāḍikās*, by the Sun's motion, and add the product to the rate of Sun's change of longitude, multiply by the Moon's daily motion and add the product to the Moon's longitude, and add it to the time of New Moon. We now get the longitude of the Sun, the longitude of the Moon and time concerned. From the Sun's longitude and the time, find the *kālalagna*. Subtract three *rāśis* from the Moon's longitude, find *carajyā*, *hārajyā*, *lambanahāraka* etc., and repeat the process till the second *lambana* is obtained. This process has to be continued till two concurrent values are obtained and the value of *lambana* is stabilized. After getting it stabilized, correct the time of New Moon with that. It gives the time of the middle of the eclipse. Then subtract three *rāśis* from *kālalagna*, find the *mahājyā* and note the sign. If it is *Tulādi* add *ākṣa*. Otherwise find the difference and assign the appropriate sign. Divide this by *natihāraka*. The result is called *nati* or parallax in latitude. Correct the latitude of the Moon with this. This is a speciality for the solar eclipse. There is no eclipse if the latitude is greater than the *samparkārdha*. The amount of obscuration is got by subtracting the latitude from *samparkārdha*. The *pramāṇa* also can be found out by dividing this by the angular diameter of the Sun.

To find the *sthityardha*, or half the duration, proceed thus. Square the latitude, subtract from the *samparkārdha* and find the root. Divide this by the difference in the rates of motion of the Sun and the Moon. This is half the duration. By subtracting and adding to the time of the middle of the eclipse, the time of the beginning and end are obtained. Find for the time of commencement, the *lambana* and *sthityardha* as before. If it is more, add it to the time of New Moon; subtract if it is less. Subtract from that the new *sthityardha*. It gives the time of commencement of the eclipse. If it is the same as the earlier figure, accept it. Otherwise repeat the process till concurrent figures are obtained. In a similar way, the time of the end of the eclipse can be decided.

Example : (1) Lunar eclipse.

We consider the Kali year 5106, called Tāraṇa. Since Rāhu is in *Meṣa* we can try this. The Full Moon day is 22nd of *Meṣa* which corresponds to 4-5-2004. We shall first find the time of opposition, when the longitudes of the Sun and the Moon differ by 180^0. We have the following data:

	3.5.2004	4.5.2004	Difference
sāyana longitude of the Sun at 5-30 A.M. I.S.T.	$43^0\,52'$	$44^0\,50'$	$0^0\,58'$
sāyana longitude of the Moon at 5-30 P.M. I.S.T.	$212^0\,02'$	$226^0\,49'$	$14^0\,47'$
Difference	$168^0\,10'$	$181^0\,59'$	

The time of opposition is

$$5^h\,30^m + \frac{11°50'}{13°49'} \times 24^h$$

$= 5^h\, 30^m + 20^h\, 34^m = 2\text{-}04$ AM on 4.5.2004

The *sāyana* longitude of Rāhu at that time $= 41^0\, 06'$

sāyana longitude of the Sun $= 44^0\, 41'$

sāyana longitude of the Moon $= 224^0\, 41'$

Longitude of the Sun – longitude of Rāhu

$$= 44^0\, 41' - 41^0\, 06' = 3^0\, 35'$$

This is less than the difference in the daily motions of the Sun and the Moon. Therefore the eclipse is to be computed.

$$\text{The latitude of the Moon} = \frac{(44°41' - 41°06') \times 4}{51}$$

$$= 17'$$

We shall find the angular diameters of the Moon and the section of the shadow.

The angular diameter of the Moon

$$= \frac{10}{251} \times 14°47' = 35'20''$$

The angular diameter of the shadow

$$= \frac{15 \times 887 - 48 \times 58}{113} = 93'$$

$$samparkārdha = \frac{35'20'' + 93'}{2}$$

$$= 64'\ 10''$$

The latitude $17' < 64'\ 10''$. Therefore the occurrence of the eclipse is confirmed.

The maximum obscuration

= *samparkārdha* – latitude of the Moon at the middle of the eclipse

$= 64' \ 10'' - 17' = 47' \ 10''$

Since it is greater than the Moon's angular diameter, the eclipse is total.

$$bimb\bar{a}ntar\bar{a}rdha = \frac{93' - 35'20''}{2} = \frac{57'40''}{2} = 28'50''$$

This is more than the latitude 17′ at the middle of the eclipse and hence eclipse is total.

The rate of change of the difference of motion/hr

$$\frac{14°47' - 58'}{24} = \frac{13°49'}{24} = 34.5$$

$$Sthityardha = \frac{\sqrt{(64'10'')^2 - 17'^2}}{34.5}$$

$$= 1^h 48^m$$

Total duration $= 2 \times 1^h 48^m$

$= 3^h 36^m$

First point of contact $= (2 - 04) - 1^h 48^m$

$= 12^h 16^m$

Last point of contact $= (2 - 04) + 1^h 48^m$

$= 3^h 52^m$

We shall find the commencement and end of totality.

bimbāntarārdha = 28′ 50″

$$vimard\bar{a}rdha = \frac{\sqrt{(28'50'')^2 - 17'^2}}{34.5}$$

$= 40^m 25^s$.

Time of *nimīlana* or commencement of totality

$= 2^h\ 4^m - 40^m 25^s$

$= 1^h\ 23^m\ 35^s$ AM

unmīlana or end of totality

$= 2^h\ 4^m + 40^m 25^s$

$= 2^h\ 44^m\ 25^s$ AM

Duration of totality $= 2 \times 40^m 25^s$

$= 1^h 20^m 50^s$

These figures are approximate. To get the exact values, the longitudes of the Moon, Rāhu, latitude of the Moon, the angular diameters of the Moon have to be calculated again for the time of first point of contact and *sthityardha* has to be again calculated. If the new first point of contact is the same as earlier, it can be accepted. Otherwise, repeat the procedure till concurrent values are obtained. This has to be done for the last point of contact, *nimīlana* and *unmīlana*. Then accurate figures can be obtained. Moreover, in the method we used many parameters are not very accurate. It may give rise to errors. The actual figures are:

First point of contact (*sparśa*) : 0 – 13 AM

nimīlana (commencement of totality): 1 – 21

Middle (*madhya*): .2 – 00

unmīlana (End of totality): 2 – 38

Last point of contact (*mokṣa*): 3 – 42

2) Solar Eclipse

We consider 26th *karkaṭaka*, Kali 5101, called Pramāthī. It corresponds to 11-8-1999. We have the following data.

We shall calculate for Thiruvananthapuram (lat. $8^0\,29'$ N, long. $76^0\,59'$ E)

	11.8.99 5-30 AM IST	12.8.99 5-30 AM IST	Difference
sāyana longitude of the Sun	$137^0\,54'$	$138^0\,52'$	$58'$
sāyana longitude of the Moon	$131^0\,51'$	$145^0\,47'$	$13^0\,56'$
Difference	$+\,6^0\,03'$	$(-)6^0\,55'$	

Difference in the rate of daily motion = $12^0\,58'$

Time of apparent conjunction = 4–42 P.M.

Sāyana longitude of the Sun = *sāyana* longitude of the Sun at conjunction = $138^0\,20'$

sāyana longitude of Rāhu = $132^0\,38'$

$$\text{Latitude of the Moon} = \frac{4\times(138^\circ\,20' - 132^\circ\,38')}{51}$$

$$= 27' \text{ (\textit{Meṣādi})}$$

We have to get the *lambana* and correct the latitude with *nati*.

$$\bar{a}k\d{s}a = \frac{pal\bar{a}\dot{n}gula \times 1287}{2}$$

$$= \frac{1.8 \times 1287}{2}$$

$$= 1158'$$

$$nat\bar{\imath}h\bar{a}raka = \frac{svade\acute{s}ah\bar{a}raka \times 50 \times 60}{81 \times 12^{\circ}\ 58'}$$

$$= \frac{3476 \times 50 \times 60}{81 \times 12^{\circ}\ 58' \times 60}$$

$$= 165'.6 \cong 166'$$

cara for the Sun's longitude

$$= \frac{222 \times 107}{138} = -172 \quad (\textit{Meṣādi})$$

$$= -2^{0}\,52'$$

prāṇakalāntara $= +\ 149' = 2^{0}\,29'$

Nāḍikās and *vinaḍikās* after sunrise at the time of conjunction is

$$\frac{(4\text{-}42 - 6\text{-}18) \times 5}{2} = 26 - 00$$

kālalagna $= 138.20 - 2^{0}\,52' + 2^{0}\,29' + 154^{0}\,12'$

$= 292^{0}\,9'$

sāyana Moon $-$ 3 *rāśis* $= 48^{0}\,52'$

carajyā for this $= 172' = 2^{0}\,40'$

$$\textit{hārajyā} = \frac{1R(1-\cos\delta)}{4\cos\varphi\cos\delta}$$

$$= .\frac{1}{4}.\frac{3438'(1-\cos 18^\circ 45')}{\cos 8^\circ 29' \cos 18^\circ 45'}$$

$$= 20.596 \cong 20.6$$

$$\frac{\textit{hārajyā}}{2} + \frac{\textit{svadeś ahāraka}}{4} = 20.6' + \frac{3476}{4}$$

$$= 20'\ 6'' + 869$$

$$= 889'\ 6''$$

Longitude of the Moon + 3 *rāśis* = $138^0 20' + 90^0 = 228^0 20'$
prāṇakalāntara for $228^0 20' = -2^0 29'$ (odd quadrant)

$$228^0 20' - 2^0 29' = 225^0 51'$$

$225^0 51'$ − *kālalagna*

$$= 585^0 51' - 292^0 09'$$

$$= 293^0 42'$$

mahājyā ($293^0 42'$) = 3167 (*Tulādi*)

mahājyā + *carajyā* = 3167 − 172′

= 2995 (*Tulādi*)

$$\textit{lambana} = \frac{2995}{889.6} = 3^n\ 22^v$$

$$= 1^h\ 21^m$$

The *lambana* is not final. It needs to be corrected by successive approximation. However, we accept it as approximate value and proceed. Since it is after noon, the correction is additive.

The correct time of New Moon with correction for *lambana* is

$4^h\ 42^m + 1^h\ 21^m = 6^h\ 03^m$

$$\text{Angular diameter of the Sun} = \frac{140}{251} \times 58' = 32'\ 21''$$

$$\text{Angular diameter of the Moon} = \frac{10}{251} \times 836 = 33'\ 18''$$

$$\textit{samparkārdha} = \frac{33'18'' + 32'21''}{2} = 32'\ 50''$$

$$\textit{sthityardha} = \frac{\sqrt{(32'50'') - 27^2}}{32.4} = 34^m\ 29^s$$

Time of *sparśa* or first point of contact $= 6^h 03^m - 34^m 29^s$

$= 5^h 28^m 31^s$

Mokṣa or last point of contact $= 6^h 03^m + 34^m 29^s = 6^h 37^m 29^s$

These figures have to be made more accurate by successive approximation, by calculating the *sthityardha* based on the longitudes of the Sun, the Moon, Rāhu and latitude at that time and repeating this till concurrent values are obtained.

Half the difference in angular diameter is

$$\frac{33'18'' - 32'21''}{2} = 28''.5$$

The latitude $27' > 28''.5$ and hence eclipse is not total.

VALANA (DEFLECTION)

31. Find the *cara* of the *sāyana* Moon at the end of *parva* (syzygy). Then find the *nāḍikās* etc. elapsed at

the time concerned (after sunrise or sunset) multiply by 6 and get in degrees, minutes etc. Correct this with the *cara* obtained earlier. Add three *rāśis* to this and find its *mahājyā* (*R* sine). Multiply by *akṣajyā* and divide by *trijyā*. Keep this figure. This is called *ākṣavalana*. Add three *rāśis* to the *sāyana* longitude of the *grāhya* (Moon during lunar eclipse and the Sun during the solar eclipse). Find its *krāṅtijyā* and the corresponding arc (This is called *āyanavalana*). Add *ākṣavalana* and *āyanavalana* if they have the same directions and take the difference otherwise. Then *valana* is obtained.

We shall discuss the rationale of the procedure.

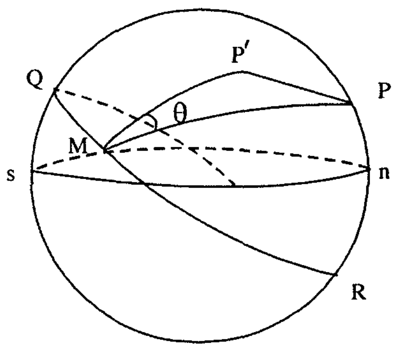


Figure 5.10

It is assumed that the Moon is on the ecliptic. In figure 5.10, M gives the position of the Moon, P and P' the poles of the equator and the ecliptic, 'n s' the horizon and QR the equator. Let $\angle PMP' = \theta$. Let ω be the obliquity. λ = the longitude of the Moon, δ = declination of the Moon.

The *āyanavalana* = $\angle PMP'$. From the spherical triangle PMP', we get

$$\cos PP' = \cos P'M \cos PM + \sin P'M.\sin PM \cos \theta$$

i.e., $\cos \omega = 0.\cos PM + 1 \sin (90^\circ - \delta) . \cos \theta$

Therefore,

$$\cos \theta = \frac{\cos \omega}{\cos \delta}$$

Then

$$\sin^2 \theta = 1 - \frac{\cos^2 \omega}{\cos^2 \delta}$$

$$= \frac{\cos^2 \delta - \cos^2 \omega}{\cos^2 \delta}$$

$$= \frac{\sin^2 \omega - \sin^2 \delta}{\cos^2 \delta}$$

$$= \frac{\sin^2 \omega - \sin^2 \omega \sin^2 \lambda}{\cos^2 \delta} = \frac{\sin^2 \omega \sin^2 \lambda}{\cos^2 \delta},$$

where λ is the longitude of the Moon.

Therefore

$$\sin \theta = \frac{\sin \omega \cos \lambda}{\cos \delta} = \frac{(R \sin \omega) R \sin (90^\circ + \lambda)}{R \cos \delta}$$

Since δ is small we get as an approximation,

$$\sin \theta = \frac{(R \sin \omega) \; R \sin (90^\circ + \lambda)}{R},$$

the expression given for *āyanavalana*.

In the case of *ākṣavalana,* the expression is only approximate. The reasoning as understood from the commentary on *Sūryasiddhānta* (Candragrahaṇādhikāra, vv.24-5) is as follows. We need the $\angle PMN$. When the body is on the horizon, it is equal to latitude (if the body is on the equator); when on the meridian it is zero. When the position is in between, using proportional parts, we get,

$$R \sin PMN = \frac{R \sin(90^\circ + \text{hour angle}) \times R \sin \phi}{R}$$

The direction of the *valana* is positive or negative according as the angle is *Meṣādi* or *Tulādi.*

One has to find the algebraic sum of arcs corresponding to the two kinds of *valana.*

DIAGRAMMATIC REPRESENTATION OF ECLIPSE

32. Draw the East - West line and from the Western end mark off a distance equal to *valana* towards the North or the South as the case may be. From that point draw a line of 65′ length parallel to the East - West line, and a similar line close to the East - West line towards the West. This shows the path of the shadow. Draw a line to represent the latitude in the concerned direction and the magnitude. This shows Moon's path. Take a point and draw a circle with that as centre and the radius of the slower entity (Moon in the case of solar eclipse and section of the shadow in the case of lunar eclipse). Subtract the amount of obscuration from the *samparkārdha*; and mark off the distance by an arrow (to show direction) and with that as centre draw a circle with the radius of the Moon's disc. The

eclipse can be visualized thus. Draw the Moon's disc in the eastern direction if it is after the middle of the eclipse and on the west if it is before the middle of the eclipse.

This gives a method for representation of an eclipse. Though it is common to both lunar and solar eclipses, it is described as though it is for the Moon. The picture would be like the following.

Lunar Eclipse

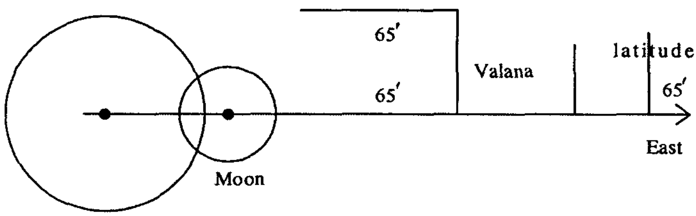


Figure 5.11(1)

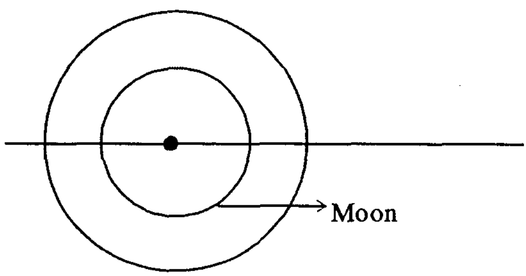


Figure 5.11 (2)

Middle of the eclipse when it is total.

The lunar eclipse starts on the eastern side and ends by the west. When the eclipse commences it is on the west of the shadow and when it ends it is in the east.

Solar eclipse is not discussed in the commentary. But it is also possible to give a representation in that case too.

COMPUTATION OF *VYATĪPĀTA*

33. Add to the longitude of the Sun twice *ayanāmśa* and subtract it from 6 *raśis* and 12 *raśis*. If the longitude of the Moon is equal to one of these, *carkrārdhadoṣa* is to be inferred. To compute *vyatīpāta*, find the *sāyana* longitudes of the Sun and the Moon.

If x is the *nirayana* longitude of the Sun and y is the *nirayana* longitude of the Moon, then either

$$x + 2a + y = 180^0 \quad \text{. . . (i)}$$

$$\text{or } x + 2a + y = 360^0 \quad \text{. . . (ii)}$$

a being *ayanāmśa*. In other words if x' and y' are the *sāyana* longitudes of the Sun and Moon respectively, then

$$x' + y' = 180° \quad \text{... (iii)}$$

$$\text{or } x' + y' = 360° \quad \text{... (iv)}$$

In either case they belong to different quadrants, but the declinations are equal in magnitude and sign in the first case, but equal in magnitude, but opposite in sign in the latter case.

Vyatīpāta is purely of astrological significance. Since astrological texts have forbidden *vyatīpāta* for fixing auspicious times, it is computed.

34. *Vyatīpāta* lasts so long as the Sun and the Moon are in the opposite quadrants (one in the odd and the other in the even quadrant), in the course of their motion. The defect is practically similar to that of eclipses.

In the case of eclipses the latitude of the Moon becomes sufficiently small to cause the eclipses. In *vyatīpāta*, the declinations become nearly equal, but the Sun and the Moon are in different quadrants. The comparison is only theoretical and no physical observation of the phenomenon is possible.

35. *Lāṭa* occurs at the star at the same distance from *Purvāṣāḍha* as *Mūla* from the star occupied by the Sun. *Vaidhṛta* occurs at the fourteenth star from that. In the *Parahita* system, the latitude of the Moon has to be multiplied by the correct rate of motion and divided by the mean rate of motion. In other respects, the method is as in the case of *Dṛk* system.

Let the Sun be in *Puṣya.* This means that the *nirayana* longitude of the Sun is between $93^0\,20'$ and $106^0\,40'$. Then *Mūla* is the twelfth star from it. *Lāṭa* occurs in the 12th from *Purvāṣāḍha* i.e., *Rohiṇī* and *vaidhṛta* occurs at *Ārdrā.* One should note that this is only a guess.

The necessity of the *Dṛk* system is also stressed. Because of the inaccuracy of the latitude calculated in the *Parahita* system, it has to be multiplied by the true rate of motion and divided by the mean rate of motion.

36. Find the latitude and the declination of the Moon, using that of the Sun. Add them or take the difference as the case may be (*infra.* v. 11). When it is equal to the declination of the Sun, *vyatīpāta* occurs, provided the

Sun and the Moon are in quadrants of different kinds. Find the angular diameters of the Sun and the Moon. Find half their sum and if the difference in the declination is less than this, *vyatīpāta* has already commenced.

The auto-commentary emphasizes that the difference of even one minute is sufficient. There are two cases. The Sun and the Moon have the same declination (equal in magnitude and sign). In that case the sum of the *sāyana* longitudes of the Sun and the Moon is 180^0. In the latter case, the declinations have the same magnitude, but different signs. The sum of the *sāyana* longitudes is 360^0 in this case. An approximate rule for guessing this is given. If the declination of the Sun is δ, then $0 \leq \delta \leq \omega$, where ω is the obliquity and the maximum is ω. This happens when the Sun is in *Mūla.*

We should consider the commencement of *sāyana uttarāyaṇa* day or winter solstice, which falls on December 22nd. It happens when the *sāyana* longitude of the Sun is 270^0. Consequently, the *nirayana* longitude is around $270^0 - 23^0 = 247^0$ at present. This is when the Sun is in *Mūla.* Equal distances from *Mūla* on either direction corresponds to equal declinations, since the winter solstice corresponds to maximum declination. Thus the rule for the guess is justified. The fourteenth from this is the day for the other *vyatīpāta.* It is called *lāṭa* when the declinations have the same sign and *vaidhṛta* when they have opposite signs.

37. If in the process of finding the declination of the Moon, it becomes necessary to subtract the declination from latitude, then the Sun and the Moon are in different kinds of quadrants. If (equivalently) the declination is less than the latitude, the same result is inferred.

The declination of the Moon is found by finding the algebraic sum of the latitude and declination of the Sun in the same position, as the Moon. If the Sun is in the northern hemisphere, its declination is *Meṣādi* and if it increases (numerically) it is *Makarādi* and if it decreases it is *Karkyādi.* Let us imagine that the Moon is also in the same hemisphere. If the declination is less than the latitude, then latitude decides the direction which is southern. This leads to a contradiction and it follows that the Moon is in southern hemisphere.

38. When there is equality of declinations of the Sun and the Moon, there is no defect if the definition does not hold good. At the same time, if the definition holds good and at the instant before or after the defined time (sum of longitudes is 180^0 or 360^0), the difference in the declination is less than half the sum of the angular diameters, there is *vyatīpāta.*

It is asserted that the two conditions necessary for the occurrence of *vyatīpāta* are :

(1) the sum of the *sāyana* longitudes of the Sun and the Moon is 180^0 or 360^0

(2) the declinations are equal in magnitude.

When these conditions hold, *vyatīpāta* is in force so long as the difference in declinations is less than half the sum of the angular diameters of the Sun and the Moon.

Half the sum of the diameters of the Sun is roughly taken as 32′ (*danta*). When the difference in declinations is less than 32′ (*dantona*), *vyatīpāta* occurs.

39,40. If the Moon is in an odd quadrant and the declination of the Moon is more than that of the Sun, then the time of equality is already over. If it is less, it is yet to come. After settling this, by knowing the result of the position of the Moon in even quadrants, proceed to find the required time. Find the latitude and declination of the Moon. If they have the same sign, add them, otherwise take the difference. This is called *vartamānadanuḥ* and is to be the divisor. Multiply the difference in the declination of the Sun and the Moon by the unit arc (225′ or 360′ as the case may be) and divide by *vartamānadanuḥ*. Correct the Moon's position with this and multiply the correction by the Sun's rate of motion (*bhāsvīya*)^2 and divide by the Moon's rate of motion and correct the Sun's position with this. Find the declination. If there is still difference, proceed further till concurrent figures are obtained, verifying at every stage, whether the position is above or below.

It is now necessary to explain the procedure for finding the time of *vyatīpāta* or the time of equality.

One can guess the star in which *vyatīpāta* is likely to occur first. Then find the *sāyana* longitudes of the Sun, the Moon and Rāhu at the beginning of the star. Find the declinations of the Sun and the Moon and note the difference. If the Sun and the Moon are in different quadrants and if declinations are equal, it happens at the time for which the results are computed, i.e., the beginning of the star (i.e., the end of the previous star). If it is less than half the sum of the angular diameters of the Sun and the Moon, the time of equality is either before or after that, which can be found out as detailed later. If they are in the

quadrants of the same kind and the Sun is in the first half, the time of equality is over and if the Sun is in the second half, the time of equality is yet to come. At this stage we have to push up or down to find the exact star. After deciding whether it is above or below, correct the longitude of the Moon by increasing or decreasing by 800' as the case may be. This is called *naraloka* correction. The Sun's longitude is increased or decreased by $800 \times \frac{\text{Sun's rate of motion}}{\text{Moon's rate of motion}}$. For Rāhu, make 1/20 of the correction for the Sun in the opposite direction. Now we get the *sāyana* longitudes of the Sun, the Moon and Rāhu in beginning of the previous star or the next star. Find the difference in declinations. If they are still in the same quadrants, make one more correction to fix the star. Thus the Sun and the Moon can be brought to positions in which they are in quadrants of different kinds and the difference in declinations is less than half the sum of the diameter of the Sun and the Moon. At this stage we have to find whether the time of equality is over or yet to come. Before the time of equality, the difference decreases and afterwards it increases. Therefore a rule can be given thus: Of the two planets, the Sun and the Moon, one is in an odd quadrant and the other in an even quadrant. If the declination of the planet in the odd quadrant is more, then it is over. If the declination the planet in the odd quadrant is less, the time of equality is yet to come. In the first quadrant declination increases and if it is more and equal to a, and the declination of the other is b, the difference is $a - b$, which increases since a increases and b decreases. In a similar way the other cases can be discussed.

We get the following rule from *Karaṇaratna* (I. 54) :

ravikrānterbhujāccandro mahāmścettad gato dhruvam |
alpaḥ koṭi śaśī tadvad viparīte viparyayaḥ ||

This means,

"If the *bhuja* of the Moon's longitude is greater than that of the Sun with the corresponding declination, then *vyātīpāta* is already over. If the *koṭī* of the Moon's longitude is less than that of the Sun, the same is the result. The reverse is the case otherwise."

This is equivalent to what we have given earlier.

Now we have fixed the star and also know whether the time of equality is earlier or later. We have at our disposal, the longitudes of the Sun, the Moon and Rāhu and the declination of the Sun and the Moon. Find the declination of the Moon by adding or subtracting the latitude and the declination from the Sun's table. This is called *vartamānadanuḥ*. Let it be v. Let the difference in declination be d_1. Find $\frac{d_1}{v} \times 225'$ or $\frac{d_1}{v} \times 360$ according as the divisions of *jyās* is into 24 or 15. Add this to or subtract from the longitude of the Moon as the case may be. For the Sun, multiply the Moon's correction by the Sun's rate of motion and divide by the Moon's rate of motion. Then, make the correction with this. For Rāhu, find 1/20 of the Sun's correction and do it in the opposite direction. Find the difference in the declinations of the Sun and the Moon. If they are still different, and the difference is d_2, find $\frac{d_2}{v} \times 225'$ or $\frac{d_2}{v} \times 360'$ as before and proceed. Continue the procedure till concurrent values are obtained.

This procedure is not followed in other books (*Pañcabodha*, V.5). The usual method is this. Find the declinations and the differences at the beginning of the star, say δ_1 and δ_2. Find $|\delta_1| + |\delta_2|$. Choose the smaller of δ_1, and δ_2. We now want the time of equality of declinations. For the Sun find

$\frac{60\times\min\{\delta_1,\delta_2\}}{|\delta_1|+|\delta_2|}$. For the Moon find $\frac{800\times\min\{\delta_1,\delta_2\}}{|\delta_1|+|\delta_2|}$ and incorporate the corrections in their longitudes. Then again find the declinations and if the equality does not occur, proceed further till equality is obtained.

The present author Śaṅkaravarman's method is slightly different from the usual method. The usual method is straightforward, successively applying the rule of three. f is a real-valued function whose domain is a sub set of R, the set of real numbers. The value of $f(t)$ is known and we have to find $f(T)$. Śaṅkaravarman, chooses some K and constructs a sequence $f(t_1)$, $f(t_2)$ . . . , $f(t_n)$, using K, such that $|f(T) - f(t_1)|$, $|f(T) - f(t_2)|$ decrease successively. This is his usual style as evidenced by his method (*infra* v.11). One cannot pronounce any opinion on this without proper analysis, except that he has devised a method of his own.

Just like eclipses one can define the beginning (*sparśa*), middle (*madhya*) and end (*mokṣa*) for *lāṭa* and *vaidhṛta* for *vyatīpāta*. There are different kinds of *vyatīpāta*.

(1) *Sampūrṇa* (Full)

If the beginning, middle and end occur for a *vyatīpāta* it is called full.

(2) *Samokṣa* (*vyatīpāta* with the end)

When it is found that the *vyatīpāta* ends in a star and on working back it is found that the Sun and Moon are in quadrants of the same kind, before reaching the middle, this is called *samokṣa*.

(3) *Sasparśa* (*vyatīpāta* with beginning)

If it is found that the difference in declination decreases when the Sun and the Moon are in quadrants of different kinds,

and before equality and in the end they come to quadrants of the same kind, then it is called *sasparśa.*

(4) *Apamocana* (*vyatīpāta* without the end)

When it happens that the Sun and the Moon are in quadrants of different kinds and the difference in declination decreases to zero, increases but before the end the Sun and the Moon come to the quadrants of the same kind, it is called *apamocana.*

(5) *Asparśa* (*vyatīpāta* without the beginning)

If it is found that the *vyatīpāta* is over and when worked back, the Sun and the Moon come to the quadrants of the same kind before reaching the middle, it is called *asparśa.*

(6) *Antarāgatam* (internal *vyatīpāta*)

If it is found that the difference in declination vanishes when they are in the quadrants of the same kind and when worked forwards and backwards they satisfy the definition of *vyatīpāta*, it is called *antarāgata*, a *vyatīpāta* without the middle. This happens when the middle is at the end of a quadrant which is also the beginning of another quadrant.

(7) *Samadhya* (with the middle)

If it happens that the Sun and the Moon have equal declinations, but when worked forwards and backwards they do not satisfy the definition, it is called *samadhya.*

(8) *Asambhava* (impossible)

When it is found that that *vyatīpāta* is ahead or over and when worked forwards or backwards, the Sun and the Moon do not satisfy the definition, it is called *asambhava.*

Example: We shall calculate *vyatīpāta* for September/ October 1991.

This corresponds to *Simha* in 1167 M.E. (Malayalam Era). Let the Sun be in *Magha*. Then *Mūla* is the 10th star. 10th star from *Purvāṣāḍha* is *Bharaṇī*. We shall calculate for *Bharaṇī* first.

(1) End of *Bharaṇī* at 3-45 PM IST on 31.8.1991

Sāyana longitude of the Moon = $50^0 24'$

Sāyana longitude of the Sun = $157^0 37'$

Sāyana longitude of Rāhu = $262^0 27'$

Declination of the Sun = $535'$

Declination of the Moon = $1130'$

Difference = $1130' - 535'$ = $595' > 32'$

Vyatīpāta has not commenced. The Moon is in odd quadrant and the Sun in an even quadrant. But the difference in declination is increasing. Therefore we shall try the previous day.

(2) End of *Aśvinī* 30.8.1991 4-08 PM IST

Sāyana longitude of the Sun = $156^0 38'$

Sāyana longitude of the Moon = $37^0 04'$

Sāyana longitude of Rāhu = $262^0 42'$

Declination of the Sun = $574'$

Declination of the Moon = $1023'$

Difference = $1023' - 621' = 402' > 32'$

The conditions are as in (1).

We try the previous day.

(3) End of *Revatī* 29.8.1991 4-10 PM IST

Sāyana longitude of the Sun = $155^0 29'$

Sāyana longitude of the Moon = $23^0 44'$

Sāyana longitude of Rāhu = $263^0 40'$

Declination of the Sun		= 613′
Declination of the Moon		= 802′
Difference	= 802′ − 613′	= 89′ > 32′

The position still continues. We shall try the beginning of *Revatī.*

28.8.1991 4-31 PM

Sāyana longitude of the Sun	= 154° 31′
Sāyana longitude of the Moon	= 10° 24′
Sāyana longitude of Rāhu	= 263° 46′
Declination of the Moon	= 605′
Declination of the Sun	= 660′
Difference = 660′ − 605′	= 55′ > 32′

But the planet with greater declination is in an even quadrant. Therefore the difference is decreasing. Moreover the Moon is still in an odd quadrant. Therefore the definition holds good. *Vyatīpāta* is yet to take place in *Revatī.*

COMPUTATION OF COMBUSTION

41. The points of combustion for the Moon onwards are 12°(*śreyaḥ*), 17°(*satya*), 13°(*gayā*), 15°(*payaḥ*), 9° (*dhana*), 15° (*śakā*) respectively. The maximum latitudes of these are 270′ (*nissāra*), 90′ (*andha*), 120′ (*nireka*), 60′ (*nīti*), 120′ (*nirayā*), 12′ (*traya*). The *pātas* are nodes having longitudes 1° 10′ (*nāyaka*), 0-20′ (*netra*), 0-20′ (*nakra*), 0-2′ (*ruk*), 2°-01 (*inaśrīḥ*), 3° 10′ (*nākula*). Subtract the longitude of the node from that of the longitude, multiply by the maximum latitude and divide by *trijyā.* The result is the latitude of the planet.

Combustion is a position when the planet comes near the Sun apparently and becomes invisible. The points at which it starts are given. They vary with the planets. If N is a node, P is a planet and D the foot of the secondary through P on the ecliptic, maximum latitude = PND. From the spherical triangle PND,

$$\sin PND = \frac{\sin(\text{lat.})}{\sin(\text{long. of the planet} - \text{long. of Node})}.$$

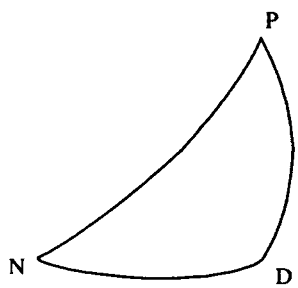
Since the angles are small sin PND is taken to be PND.

$R \sin PND = PND$ in minutes

Therefore we get,

[3]Latitude = Max. latitude × sin (long. of the planet – long. of Node)

$$= \frac{\text{Max.lat.}\left[R(\sin \text{long. of the planet} - \text{long. of Node})\right]}{R}$$



42. Subtract the longitude of the Node from *śīghrocca* for Mercury and Venus, from the third longitude in the case of Mars, Jupiter and Saturn and from true longitude in the case of the Moon. Find its *jyā*, multiply by the maximum latitude and divide by

trijyā. In the case of the Moon it is the accurate latitude. For others subtract the longitude of Node from *śīghrocca*, find the lunar *jyā* and multiply the earlier figure by that. Another method is to subtract *śīghrocca* from the penultimate longitude find the lunar *jyā* and divide latitude obtained by that.

43. In *Parahita* system, the method is to subtract the longitude of the node from the longitude corrected by *mandaphala* and then find the latitude. Then note the *jyā* segment in the last *śīghra* correction. Then add to or subtract from this, 225′ according as the last *jyā* segment referred to is *Karkyādi* or *Makarādi.* Multiply by the latitude and divide by 225′.

The term *mandasphuṭa* has been defined in the auto-commentary as follows: When the mean position is corrected with *mandasaṃskāra*, the result is called *mandasphuṭa.* For Jupiter, Mars and Saturn correct with *mandaphala.* For Venus and Mercury find *śīghrocca* and correct it by *mandaphala.* In either case find the last *śīghrajyā* segment and note whether it is *Karkyādi* or *Makarādi.*

In works like *Pañcabodha* there is some difference. The latitude needs to be corrected and different methods are followed.

First the author suggests finding the latitude and dividing by lunar *jyā.* In this stanza that is not emphasized. Nor does the auto-commentary explain this with clarity. It appears that there are three different ways of finding the correct latitude of the Moon according to the author of the present work.

It is mentioned in stanza 42 that the accurate latitude can be obtained by multiplying the lunar *jyā* arising out of the value obtained by subtracting *śīghrocca* from the true longitude, by the latitude obtained earlier. It is also mentioned that accurate

latitude can be obtained dividing the latitude by the lunar *jyā* arising out of the value obtained by subtracting *śīghrocca* from the penultimate value of the longitude. The true and the penultimate longitudes do not differ much. The *śīghrocca* is only the mean Sun. Thus the lunar *jyā* can be the maximum possible. In one case we divide by that and in the other case we multiply that. How can both be true? Multiplication may lead to an inflated figure. The maximum *naronvādijyā* is *pannagaḥ* (301′). Division sounds more appropriate. Even the auto-commentary is not helpful here.

44, 45. Find the *sāyana* longitudes of the planet and the Sun. When the Sun's longitude is less, find the *sāyana* longitude of the planet and *kālalagna* in the morning. When the planet's longitude is less, do it for the sunset. Find the latitude of the planet and find the sum or difference of this and the declination, as required. Using this declination, find *cara* of the planet. It can be found out using the figures for *Lokamalayārkāvu,* dividing by 692 (*rāddhānta*) and multiplying by the *phalabhā* of the place. Subtract three *rāśis* from the planet's longitude, find its latitude multiplying by the lunar *jyā* and dividing by 674 (*vasanta*). Add to or subtract from the longitude of the planet as the case may be. This is called *darśanasamskāra.* Correct the *kālalagna* with *cara* and add six *rāśis* and correct with *cara* in the opposite sign in the evening. Correct the planet's position with *cara* and *prāṇakalāntara.* If the difference between *kālalagna* and the planet's position is less than the bound for combustion, the combustion has already started. Otherwise, it is yet to commence.

The exact time can be obtained by rule of three.

Combustion or *mauḍhya* occurs when the planet apparently comes near the Sun. The difference in the longitudes (nearly) of the planet and the Sun is given in the beginning of the section. The beginning of *mauḍhya* is called *astamaya* (heliacal setting) and the end is called *udaya* (heliacal rising). For Mercury and Venus combustion occurs twice, in the course of their synodic periods (synodic period is the time taken by the body to revolve once in the sky relative to the Sun). During direct motion it occurs once and occurs again in retrograde motion. For Mercury the combustion during direct motion lasts for nearly 32 days. After this, the combustion during retrogression starts about 34 days later and lasts for about 16 days and direct motion once again starts after about 34 days. For Venus also the phenomenon is similar. For Mars, Jupiter and Saturn combustion occurs for about a month once a year. For Mars, Saturn and Jupiter the beginning of combustion is calculated for sunset and end for sunrise. For Mercury and Venus, the same procedure is adopted during direct motion and it reversed during retrogression. For the Moon, the beginning is calculated for sunrise and the end for sunset.

ŚṚṄGONNATI (ELEVATION OF THE MOON'S HORNS)

46. In the computation of *śṛṅgonnati* all calculations of moon's shadow, have to be done in *Dṛk* system after getting the second Moon. The latitude of the Moon is not required.

In the following verse the phases of the Moon are found out, when less than half of the lunar disc is illuminated (*Sūryasiddhānta*, 20.10.14 comm.)

47. Find the longitude of *tithi*, multiply the latitude of the Moon by the smaller of *R* sine and *R* cosine of *tithisphuṭa* and divided by the larger. This is called *vikṣepa valana*. If the longitude of the *tithisphuṭa* is in the odd quadrant, this has the same direction as *vikṣepa*. Otherwise, it is the opposite. Subtract the longitude of the second Moon from *kālalagna* increased by three *rāśis*, find its *R* sine, multiply by *ākṣavalana*. The sign depends on that of the *R* sine.

Tithisphuṭa is obtained by subtracting the *sāyana* (*nirayana*) longitude of the Sun from that of *sāyana* (*nirayana*) longitude of the Moon. In this section, *sāyana* longitudes are defined. But it is mentioned in the commentary that the *sāyana* Sun is to be subtracted from the second Moon to get *tithisphuṭa*. Throughout the section only the second Moon is used.

48. Find the *R* sine of the declination of the Moon corresponding to the longitude increased by three *rāśis* and find the *āyanavalana*. Find the algebraic sum of arcs of *ākṣavalana*, *āyanavalana* and *vikṣepa valana*. Find its *R* sine, the angular radius of the Moon and multiply by that. Divide the former result by the latter. The result in minutes gives *śṛṅgonnati*.

49. Subtract the Sun's longitude from that of the Moon, find its *R* cosine. Subtract from *R* if it is *Makarādi* and add to it if it is *Karkyādi*. When it is multiplied by the angular radius of the Moon and divided by *trijyā*, the *sitamāna* or measure of the white part is obtained. Find the difference between *sitamāna* and the radius of the Moon. This is called *śara*. Square

the angular radius of the Moon and divide by *śara* and add it to *śara*. Half of this is called *sūtra*.

Thus we have five important components – the radius of the Moon's disc, *śṛṅgonnati* in angular measure, *sitamānāṅgula*, *śarāṅgula* and *sutrāṅgula*.

50. Mark the East – West line. From the radius showing the direction, measure in the concerned direction, according to the nature of the Moon (waxing or waning), a length equal to *śṛṅgonnati*, and mark it; in the opposite direction mark a point at the extremity of the diameter passing through the centre. Mark a point inside that, at a distance equal to *sitamāna* and draw a circle passing through the three points.

The auto-commentary is vague and does not give the procedure with clarity. The idea is to represent the Moon's horns[4]. We can guess from the verse that the method is quite similar to the one in *Tantrasaṅgraha* (VIII. 26-35) which we detail below :-

Draw a circle with the compass with radius equal to the angular radius of the Moon to represent it. Let M be the centre. Draw two lines EW and NS through M to represent the East - West and North - South line. These divide the lunar disc into four parts. Then mark in the direction of the Sun a point P on the circumference at a distance (measured by the chord) equal to *śṛṅgonnati* from the west point in the side occupied by the Sun. Mark another point Q diametrically opposite to that at the same distance from the east point and draw the diameter joining them. Then draw circles with these two as centres and draw a line joining the points of intersection. This is called *tiryagrekhā*. Then measure a distance from the bottom of the *tiryagrekha* (vertical line) equal to *sitamāna* and mark the point R. Draw a

circle passing through the three points. Then we get the picture of the Moon as we see below.

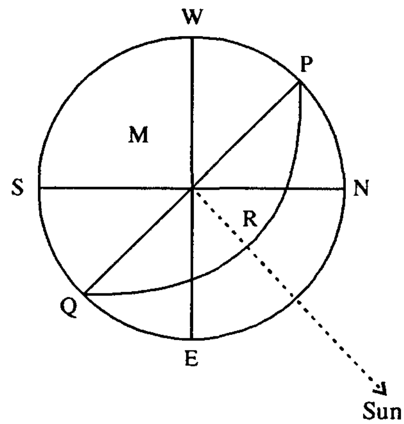


Figure 5.12

51. By the grace of Śrī Lokāmbā, the methods of *Pañcabodha* have been described by me. Those who read this realize the quintessence of the well-known mathematical methods. Let Lord Kṛṣṇa who manifests as direction and time and whose sport manifests as time and mathematical knowledge, enhance our prosperity.

In the concluding verse of the chapter he pays obeisance to Śrī Lokāmbā, the tutelary deity of the family. He also prays to Lord Kṛṣṇa to shower prosperity. He observes that Lord Kṛṣṇa manifests as time and direction (time and space, more explicitly) and all the heavenly movements are caused by His will. He is the creator and the material cause of the Universe (*upādāna* and *nimitta kāraṇa*)[5].

NOTES

1. S. Madhavan, *Models in Indian Astronomy* (National seminar on Indian Intellectual Tradition, Sree Sankaracharya University of Sanskrit, 2004).

2. The term '*bhāsvīya*' used in the sense of "relating to the Sun". It can be explained thus: *bhā svam yasya sa bhāsvaḥ sūryaḥ* (one whose wealth is radiance is the Sun). *Bhāsvīyam* means relating to the Sun using the *sūtra* – *vṛddācchaḥ* (*Aṣṭādhyāyī*, IV.2.114).

3. See Chapter VI for details regarding latitude.

4. When less than half the lunar disc is illuminated, it appears to have horns. This is from *ekādaśī* of *Kṛṣṇapakṣa* to *pañcamī* of *Śuklapakṣa* (Eleventh *tithi* of the dark half to the fifth *tithi* of bright half). (See *Pañcabodha,* 8.1, comm.). *PQEN* is the illuminated part. See fig. 5. 12 related to *Kṛṣṇapakṣa*. For *Śuklapakṣa* corresponding changes are needed.

5. The opening verse of Nīlakaṇṭha Somayajin's *Tantrasaṅgraha* worships Viṣṇu, who is the cause of the Universe and the Supreme Effulgence :

   he viṣṇo nihitam kṛtsnam jagattvayyeva kāraṇe |
   jyotiṣām jyotiṣe tasmai namo nārāyaṇāya te ||

   The creation of the Universe is his sport, as has been observed by Vedāntadeśika in his *kāvya, Yādavābhyudaya* (1.3) thus:

   krīḍātūlikayā svasmin kṛpārūṣitayā svayam |
   eko viśvamidam citram vibhuḥ śrīmānajījanat ||

   "The unique omniscient Lord who is always with goddess Lakṣmī, painted the variegated Universe on Himself (or Universe, which is a picture), with His sport as his brush, smeared with mercy".

   That Time is a manifestation of the Supreme Being is a well-known concept. The idea is contained in the following verse from *Yādavābhudaya* (VIII.2):

   anekarūpaiḥ svayamekarūpaḥ kālātmakam rūpamakālakalyaḥ |
   ṛtuprabhedairanubhūya reme rāmāsakho rāmamanuprayātaḥ ||

   "One who has himself a unique form, and who is not affected by Time assumed the form of Time with several forms caused by the change of seasons and enjoyed himself with lovely women following Rāma (Balarāma)".

The content is that Lord Kṛṣṇa assumed the form of Time with seasonal changes and made the lovely women and Balarāma happy. Appayya Dīkṣita, while commenting on the verse observes thus:

"*tataḥ kālātmako yo'sau tavāmśaḥ kathito hareḥ*" – *ityādau bhagavataḥ kālarūpabhedatvam prasiddham* |

This means that from the statement 'Then, that which has the true form of Time, is thy manifestation, O! Hari' the Lord's assumption of the form of Time is well known.

It is also said that Time is the one with form and without form, that performs the cosmic functions of creation, protection and destruction as evidenced by *Maitrāyaṇyupaniṣad* (VI.14) :

kālāt sravanti bhūtāni kālād vṛddhim prayānti ca |
kāle cāstam niyacchanti kālo mūrtiramūrtimān ||

CHAPTER VI

PARAHITAGAṆITA

As the title indicates, this chapter deals with the *Parahita* methods of computation. This chapter also discusses a further revision given in Kali 4708 (1607 A.D.). The need for periodical revision of astronomical constants is also stressed.

ĀRYABHAṬA'S FIGURES FOR THE *BHŪDINA*, REVOLUTIONS ETC.

1. The astronomical treatise written by Āryabhaṭa in Kali 3623 (522 A.D.) gave well-nigh accurate results, whereas the *Siddhāntas* of Brahmā and others became inaccurate. In this system, the number of civil days in a *caturyuga* is 1577917500 (*nṛnamatsatkeḷisārthaśayaḥ*). The number of revolutions of the Sun, Mercury and Venus is 43, 20,000.

   The length of the solar (sidereal) year

   $$= \frac{1577917500}{4320000} = 365\ .\ 2586805 \text{ days}$$

   The modern figure is 365.2563604

2. The number of revolutions of the Moon is 57753336 (*ṣaḍbalaguṇasusṛṇiḥ*); of Saturn is 146564 (*śvetamattebhapatnī*); of Jupiter, is 364224 (*viprendro vṛttilagnaḥ*); of Mars, is 2296824 (*jvaradiṣudhikharaḥ*); and of the Moon's nodes (Rāhu and Ketu) is 232226 (*citrarekhāmbaraḥ*), of the apogee of the Moon, is 488219 (*dhikkuru hṛdagham*); of

śīghrocca of Mercury, is 17937020 (*jñaśrīnatho buddhisevyaḥ*) and of the *śīghrocca* of Venus, is 7022388 (*hṛdigururinasūḥ*). For Mars, Saturn and Jupiter, the *śīghrocca* is the same as that of the mean Sun.

PARAHITA SYSTEM

3. The number of revolutions of a planet (in a *mahāyuga*) multiplied by the number of Kali days elapsed and divided by the number of days in the *mahāyuga* gives the number of revolutions elapsed. The remainder, when divided successively by 12, 60, 60 gives the mean position in *rāśis*, degrees and minutes.

In the year Kali 3785 (683 A.D.), it was decided by wise men to introduce the correction called *Parahita* (in the interest of others) for all planets other than the Sun, in view of the difference between the observed and computed positions.

The first part of the stanza gives the method of finding the mean position of a planet. Accordingly, the mean position

$$= \frac{\text{No. of Kali days elapsed} \times \text{No. of revolution s in } mah\bar{a}yuga}{\text{No. of days in } mah\bar{a}yuga}$$

For example, we shall find the mean position of the Moon when 186000 days in Kali are over. The mean position of the Moon $= \frac{186000 \times 57753336}{1577917500} = 6807.78335749$ *bhagaṇas* (approximately) when reduced to minutes this works out 12920′.521784.

In the second half it has been observed that *Parahita* system was introduced in the year 3785 Kali (683 A.D.). The

author has also observed that *Āryabhaṭīya* was composed in the year Kali 3623 (A.D. 522). The basis is the following *Āryabhaṭīya* (III.10):

ṣaṣṭyabdānām ṣaṣṭir
yadā vyatītāstrayaśca yugapādāḥ |
tryadhikā vimśatirabdā
stadeha mama janmano'tītāh ||

This can be interpreted in two different ways -

(i) 'For me who was born when three quarters of a *mahāyuga* (*Kṛta, Treta* and *Dvāpara*) and 3600 years were over, twenty-three years were over at the time of composition of the work'.

(ii) 'When three quarters of the *mahāyuga* (*Kṛta, Tretā* and *Dvāpara*) and 3600 years of Kali were over, 23 years were over since my birth'.

According to the first interpretation, the work was written in 3623 Kali, when Āryabhaṭa was 23 years of age. The second suggests that he completed 23 years of age in 3600 Kali.

Varāhamihira's reference to 427 *śaka* (505 A.D.) in *Pañcasiddhāntikā* (I.11) indicates that it was written around 505 A.D. or after that. But Varāhamihira in his (*Pañcasiddhāntikā* XV. 20) refers to Āryabhaṭa and consequently this work can be placed only after the date of composition of *Āryabhaṭīya*.

Even when Āryabhaṭa introduced the system, there were some shortcomings. As days advanced, the errors manifested in a spectacular way and the assembly of scholars who met in Tirunāvay in 683, at Māmāṅkam (*mahāmāgha*), a twelve-yearly festival, promulgated the new system of *Parahita* to rectify the Āryabhaṭan system. Haridatta's (650 - 700 A.D.), *Grahacāra*

nibandhana (XV. 20) deals with the *Parahita* system. A correction introduced therein was the *śakābda samskāra* or *bhaṭa samskāra.*

Whatever be the date of composition of *Āryabhaṭīya*, the correction in the opinion of the scholars was only from 3623 Kali.

ŚAKĀBDA CORRECTION

4. From the Kali year subtract 3623 (*gotratuṅga*). Keep the figure. Multiply it by 420 (*nirūḍhī*) and divide by 235 (*māgara*). This is in minutes to be added to the *śīghrocca* of Mercury. Multiply the figure retained by 20 (*nakha*) and divide by 235 (*māgara*). This is minutes. Convert into degrees etc. and add it to the mean longitude of Saturn. Multiply the figure by 45 (*śubha*) and divide by 235 (*māgara*). This is in minutes. Convert into degrees etc. and add it to the mean position of Mars. Multiply the figure by 47 (*sāvana*), divide by 235 (*māgara*). This is in minutes to be subtracted from the mean position of Jupiter. Multiply the figure by 153 (*gaṇaka*) and divide by 235 (*māgara*). This is in minutes to be subtracted from the *śīghrocca* of Venus. Multiply the figure by 9 (*dhī*), divide by 85 (*mada*) and subtract from the mean Moon. Multiply the figure by 65 (*śānti*), divide by 134 (*vilaya*) and subtract from the *mandocca* of the Moon increased by three *rāśis.* Multiply by 13 (*śloka*), divide by 32 and subtract from Rāhu's position, decreased by 6 *rāśis.* These are the correct *Parahita* positions.

Because the Aryabhatan system did not give accurate results, a *śakābda* correction was introduced as indicated earlier.

The Sun is excluded from corrections. From the Kali year subtract 3623. Let the remainder be r. Multiply r by $\frac{9}{85}$ and add it to Moon's *madhyama* (mean longitude). Multiply r by $\frac{65}{134}$ and subtract from the Moon's *mandocca* increased by 3 *rāśis* and so on, using the multiplicants:

$$-\frac{9}{85}, -\frac{65}{134}, -\frac{13}{32}, +\frac{420}{235}, +\frac{45}{235}, \frac{47}{235}, -\frac{153}{235} \text{ and } +\frac{20}{35}$$

Moon's *mandocca* increased by 3 *rāśis*, Moon's node decreased by 6 *rāśis*, Mars, *śīghrocca* of Mercury, Jupiter and *śīghrocca* of Venus and Saturn respectively.

Haridatta suggests in his *Mahāmārganibandhana* (III.44) these corrections. But according to him 444 has to be subtracted from the Śaka year. Since Śaka year is lunar and Kali year is solar, there is some difference though not significant.

5. Find the *sāyana* longitude of the Sun and the *cara*. For finding the duration of the day, the whole of *cara* has to be used. For other times, find the number of *gaṭis* etc. elapsed. If it exceeds 15 subtract 15. Otherwise retain it as it is. Multiply this by *cara* and divide by 15. The sign of *cara* during night is opposite to that in day. There is no *cara* at noon or midnight.

Cara is the ascensional difference and sin *cara* = tan φ tan δ, where φ is the latitude of the place and δ is the declination of the Sun.

ŚAKĀBDA CORRECTION AT THE DESIRED TIME

6. To get the positions of planets at a place with latitude φ (>0) one has to find the *deśāntara* correction, Sun's *mandaphala*, and *cara* for the *sāyana* Sun and get the position of the Sun at sunrise. By multiplying the time

elapsed since the sunrise by the rate of motion and adding to it the longitude (mean or true) at sunrise the longitude (mean or true) of the planet at the desired time is obtained. On the other hand, to get the moments of beginning or end of *tithi*, the corrections, *deśāntara, cara, prāṇakalāntara* etc., are to be done in the opposite direction (*tulāmśa*).

While finding the mean positions as described earlier, the mean positions of the planet at the rising of the mean Sun at Laṅkā are obtained. To get the positions at a place of latitude φ (>0), corrections have to be effected. We need first of all the true Sun. This is done by adding the *mandaphala. Deśantara* correction is effected to get the longitude corresponding to the (terrestrial) longitude of the place. *Cara* correction is for adjusting to the sunrise of the place. For any planet, the time elapsed since sunrise has to be multiplied by the rate of motion and used for getting the position at the desired time. For finding the mean longitude of the planet mean rate of motion should be used and for finding true longitude, the true rate of motion has to be used. *Thithisphuṭa* is obtained by subtracting the longitude of the Sun from that of the Moon. Since the times for fixed longitude have to obtained, the corrections have to be reversed in sign.

7. The computation of the position of a planet at a time can be done by this method or many other methods. But the *prāṇakalāntara* of the diurnal duration as given by *Parahita* system is to be avoided.

The auto-commentary explains that the corrections can be effected to the mean position and true position can be obtained. One can also find the true position and effect the corrections.

MANDOCCAS (APOGEES) OF PLANETS

8. The *mandocca* of the Sun is $2^r 18^0$ (*daityāriḥ*). Those of Mars, Mercury, Jupiter, Venus and Saturn are respectively $3^r 28^0$ (*jarāgaḥ*), $7^r 0^0$ (*nānārtha*), $6^r 0^0$ (*ananta*), $3^r 0^0$ (*anaṅga*) and $7^r 26^0$ (*ṣaḍrasa*).

Mandocca or the apex of slow motion corresponds to Apogee or Aphelion. Except for the Moon, it is a point which is practically fixed.

These figures tally with those given in *Pañcabodha* (IV.2). The figures are precisely the longitudes of *mandoccas.*

MANDAPARIDHIS AND *ŚĪGHRAPARIDHIS*

9. The first and last values of *paridhis* (circumferences) of *mandvṛttas* for Mars, Mercury, Jupiter, Venus and Saturn are respectively, 14 and 18, 7 and 5, 7 and 8, 4 and 3 and 9 and 13.

For calculating the *mandaphala* of a planet the procedure is this. If ℓ is the mean longitude, and K is the *mandocca*, the mean anomaly is $\ell - K = m$ (say). Then

the *doḥphala* (or *mandaphala*, since the arc is small)

$$= \frac{a}{80} \times (R \sin m) \text{ in minutes,}$$

where $R = 3438$. It is for the computation of *doḥphala*, *mandaparidhi* is given.

In Aryabhatan school, the *mandaparidhi* is not fixed for Mars, Mercury, Jupiter, Venus and Saturn. They have variable values. The values given correspond to $0 - 90^0$, $90^0 - 180^0$. These are repeated for $180^0 - 270^0$ and $270^0 - 360^0$. So one has

to calculate the *mandaparidhi*, for a given value of m. The procedure is given in stanza 11below.

In *Sūryasiddhānta* (II.18), the Sun and Moon have variable *mandaparidhis*. This amounts to the orbit consisting of two ecliptic arcs, a unique feature of the *Siddhānta*. This has been studied in a paper of S. Madhavan[1].

10. The first and last values of *śīghraparidhis* of Mercury, Jupiter, Venus and Saturn are 53 and 51, 31 and 29, 16 and 15, 59 and 57 and 9 and 8 respectively.

The five planets other than the Sun and the Moon require a *śīghra* correction also. For this purpose, the circumferences of the *śīghravṛttas* are given.

FINDING THE ACCURATE CIRCUMFERENCES OF *VṚTTAS* (EPICYCLES)

11. To find the accurate value of the circumference, find the difference between the two values multiply by $R \sin m$, where m is the mean anomaly, and divide by R. Add to or subtract from the second value according as the required value is less or more than the first value.

This requires some explanation. If the first value is a_1 and the second value is a_2 then the accurate value is a_1 if $m = 0$, equal to a_2, if $m = 90^0$ equal to a_1 when $m = 180^0$ and equal to a_2 when $m = 270^0$. Find $|a_2 - a_1| R \sin m$ and divide by R. The result is $|a_2 - a_1| \sin m$. Add it to or subtract from a_2 according as $a_1 < a_2$ or $a_1 > a_2$. The correction is always for a_1.

12. The accurate circumferences of the *mandavṛttas* of the Sun and the Moon are respectively 3 (*gānam*) and 7 (*sūnam*). The *mandaphala* is obtained by

multiplying the accurate circumference by R sine of *mandakendra* and dividing by 80.

Let ℓ be the mean longitude of the Sun and K the longitude of *mandocca*. Then the *mandakendra* or mean anomaly $= m - \ell - K$. *Mandajyā* $= (R \sin m)$ and *mandaphala* $= \frac{3}{80}$ $(R \sin m)$.

Strictly speaking, *mandaphala* $= (R \sin)^{-1}$ (*mandaphala*). Since the value is usually small, it is equal to R sine of the arc in minutes approximately and the difference can be neglected.

13. Find the R sine and R cosine of *śīghrakendra*. Multiply them by the *śīghraparidhi* and divide by 80. These are the *doḥphala* and *koṭiphala*. R cosine is negative if *śīghrakendra* is between 90^0 and 270^0 (*Karkyādi*) and positive if it is between 270^0 and 90^0. Add the *koṭiphala* to R or subtract from it accordingly. Square *doḥphala* and this figure, add them and extract the square root. The result is the *śīghrakarṇa*. Then $R \sin(sighraphala) = \dfrac{dohphala}{sighrakarṇa} \times R$ and the arc corresponding to is as *śīghraphala*.

The above gives the method of finding *śīghraphala*. Let the *śīghra paridhi* be a and let the *śīghra kendra* be m. Then,

$$dohphala = \frac{a}{80}(R \sin m)$$

$$sphuṭa\ koṭi = R + \frac{a}{80}(R \cos m)$$

$$śīghrakarṇa. = \sqrt{\left(R + \frac{a}{80} R \cos m\right)^2 + \left(\frac{a}{80} R \sin m\right)^2}$$

$$= \sqrt{R^2 + \frac{a^2}{80^2} . R^2 + 2 . \frac{a}{80} . R \cos m}$$

$$\textit{śīghraphala}\ \text{ɜ} = (R \sin)^{-1} \left[\frac{R . a}{\textit{sighrakarṇa}} \times \frac{R \sin m}{80} \right]$$

14. Convert the *mandaphala* in the form of arc into minutes. Find the R sine of that and multiply by 80 and divide by the R sine of *mandakendra*. The result is the accurate *mandaparidhi.*

$$\textit{Mandaphala} = (R \sin)^{-1} \left[\frac{a}{80} (R \sin m) \right] = K \text{ (say)}$$

Then $\frac{a}{80}$ R sin $m = R$ sin K.

$$a = \frac{80\, R \sin K}{R \sin m}$$

15. From the arc of *śīghra kendra* reduced to the first quadrant, subtract the *śīghra phala* if it is *Makarādi* and add to it if it is *Karkyādi.* Find its R sine and divide the *śīghrajyā* multiplied by 80. The result is *śīghra paridhi.*

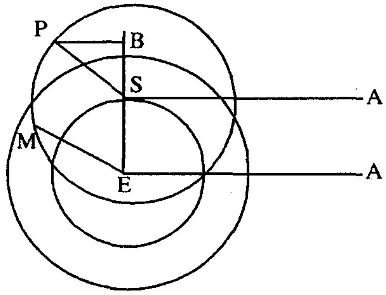


Figure 6.1

In figure 6.1, E represents the Earth, A is *Meṣādi*, S is *śīghrocca*, in *śīghravṛtta* and M mean position in *kakṣyavṛtta* and P the planet in *pratimaṇḍala*. Draw PB perpendicular to ES. SP is parallel to EM. in view of the theory of eccentric circle for planets,

Mean longitude $= \angle AEM = \angle ASP$.

Also $\angle AES = \angle ASB =$ long. of *śīghrocca*.

Therefore,

śīghra kendra = Mean longitude − *śīghrocca*

$= \angle ASP - \angle ASB = \angle BSP = \angle BEM.$

From Δ SEP, we get

$$\frac{R \sin SEP}{SP} = \frac{R \sin SPE}{ES}$$

Therefore

$$ES = SP \times \frac{R \sin SPE}{R \sin SEP}$$

$\angle SEP = \angle SEM - \angle PEM =$ *śīghra kendra* − *śīghraphala*

= *śīghra kendra* ~ *śīghraphala* (numerically)

Also R sin SPE is *śīghrajyā*

Instead of multiplying by $SP = R$, if we multiply by 80, we get the *śīghraparidhi*.

16. The circumference of the last value of the *paridhi* corresponds to the last *jyā*. The first value of the *paridhi* is got by correcting the last value by twice the correction corresponding to one *rāsi*.

If a, a_2 are the initial and final values corresponding to the *paridhis*, those corresponding to the end of odd and even quadrants then,when the *kendra* is m,

$$paridhi = a_2 \pm \frac{|a_1 - a_2||R \sin m|}{R}$$

$$= a_2 \pm |a_1 - a_2| \sin m$$

according as $a_2 < a_1$, or $a_2 > a_1$, when $m = 90^0$,

paridhi $= a_2 \pm |a_1 - a_2| = a_1$ Also, when $m = 30^0$, $R \sin m = R/2$ and we get the result.

METHODS OF FINDING LONGITUDES OF PLANETS

17. The method for computing the longitudes of Saturn, Jupiter and Mars is as follows. Find the mean longitude of the planet, subtract *mandocca*, and find the *manda phala* and correct the mean position with half *manda phala.* Subtract *śīghrocca* find the *śīghra phala* and correct the value with half *śīghraphala.* Subtract *mandocca*, find *manda phala* and correct the mean longitude with that. Then subtract *śīghrocca* and effect *śīghra* correction. The figure gives the true longitude of the planet.

It is observed in the auto-commentary that the mean longitude should be corrected for the place concerned before computation. *mandajyā* is positive if the *manda kendra* lies between 180° and 360° (*Tulādi*) and negative if *manda kendra* lies between 0 and 180° (*Meṣādi*). This method is given in *Pañcabodha* (III.4) also. Though the *manda* and *śīghra* corrections are effected twice in general there is

variation in the order and the amount. *Mandoccas* are available in texts, *śīghrocca* for Mars, Saturn and Jupiter is the mean Sun.

18. In the case of Mercury and Venus all these have to be done, except the first correction. The mean position of Mercury and Venus is that of the mean Sun.

The procedure for computing the true longitudes can be given thus: Find the mean Sun (which is the mean position for Mercury and Venus), subtract *śīghrocca*, find *śīghraphala* and correct the mean Sun with half the *śīghraphala*. Then subtract *mandocca*, find the *manda phala* and correct the mean sun. Then subtract *śīghrocca,* find *śīghraphala* and correct the earlier figure. The result is the true longitude.

The planetary motion is based on the theory of epicycles. In the case of the Sun and the Moon it has been described earlier. In the case of star planets it is more complicated. We need to include *śīghra vṛtta* and the correction arising out of that. First, we shall examine the rationale of the correction.

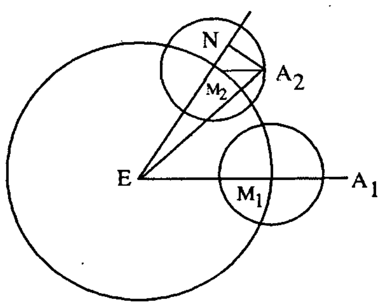


Figure 6.2

In figure 6.2, let E be the centre of the *kakṣyāvṛtta*, M_1, the position of *mandocca* and A_1 that of the planet in *manda vṛtta*

when their longitudes are equal. After some time M_1 comes to M_2 and A_1 comes to A_2 in the *mandavṛtta* and $\angle M_1EM_2 = \angle A_2M_2N$, where N is the foot of the perpendicular from A_2 to EM_2.

The correction required = $\angle A_2EN$. It is positive or negative according as the angle is *Tulādi* or *Meṣādi*. We take the radius of *kakṣyāvṛtta* to be R.

The correction required to get the true longitude from the mean longitude $\angle A_2EM_2 = m$. We can take the arc R sin $m = A_2N$ as first approximation.

$$\sin m = \frac{A_2N}{M_2A_2}$$

We shall find A_2N

$$A_2N = (M_2A_2) \sin m$$

The correction required = $\angle A_2EN$ where:

$$\sin A_2EN = \frac{A_2N}{EM_2} \text{ (approximately)}$$

mandaparidhi : *kakṣyāparidhi* $= a : D$

Therefore

$$R \text{ sine of the correction} = R.\frac{A_2N}{EM_2} = \frac{M_2A_2\ R\sin m}{EM_2}$$

$$= \frac{a}{D}(R\sin m) \text{ (in minutes).}$$

Actually, $M_2N = \sqrt{M_2A_2^{\,2} - A_2N^2} = \sqrt{\frac{a^2}{D^2}R^2 - \frac{a^2}{D^2}R^2 sin^2 m}$

$$= \frac{a}{D}.R\cos m.$$

Thus we get

$$EA_2 = \sqrt{EN^2 + A_2N^2}$$

$$= \sqrt{(R + \frac{a}{D}R\cos m)^2 + (\frac{a}{D}R\sin m)^2}$$

This is called *manda karṇa*. This is however called *vyastakarṇa* in the text (see VI.21).

Śīghrakarṇa can be found similarly. Thus we get

$$R\sin m = \frac{R}{mandakarṇa} \times doḥ\ phala$$

Since *manda paridhi* is generally small, it is taken as *doḥ phala* and even the angle in minutes can be taken to be equal to *doḥ phala*. In the case of *śīghra* correction, we take the

$$śīghraphala = (R\sin)^{-1}.\frac{R\ doḥphala}{śīghrakarṇa}$$

Thus we can get *mandaphala* and *śīghraphala* using *manda* and *śīghra* corrections. While finding *karṇa* we get $R + \frac{a}{D}(R\cos m)$ or $R - \frac{a}{D}(R\cos m)$ according as m is *Makarādi* or *Karkyādi*. The real problem is the mention of four corrections. Different works give these things in different ways, as detailed below :

Work	Corrections for Mars, Jupiter and Saturn				Corrections for Venus and Mercury			
1. *Āryabhaṭīya*	½ *m*	½ *s*	1 *m*	1 *s*	-	1 *s*	1 *m*	1 *s*
2. *Sūryasiddhānta*	½ *s*	½ *m*	1 *m*	1 *s*	½ *s*	½ *m*	1 *m*	1 *s*
3. *Siddhānta śiromaṇi*	1 *m*	1 *s*	1 *m*	1 *s*	1 *m*	1 *s*	1 *m*	1 *s*
4. *Pañcabodha*	½ *m*	½ *s*	1 *m*	1 *s*	-	½ *s*	1 *m*	1 *s*
5. *Sadratnamālā*	½ *m*	½ *s*	1 *m*	1 *s*	-	½ *s*	1 *m*	1 *s*
6. *Pañcasiddhāntikā* (*Saurasiddhānta*)	½ *s*	½ *m*	*m*	*s*	½ *s*	½ *m*	*m*	*s*

s - *śīghra* correction ; *m* - *manda* correction.

In *Siddhāntaśiromaṇi* (III. 35), the position of Mars has to be computed with ½ *m* and ½ *s* and 3rd and 4th steps have to be repeated till concurrence is obtained.

In *Pañcasiddhāntikā* (XVII. 10-11ab), additional corrections are given for Venus and Mercury.

In fact, a *śīghra* correction and a *manda* correction should suffice. Because of the inadequacy of the theory the procedure was made elaborate with a view to achieving accurate results. Different methods of experimentation have lead to this discord.

The mean position can be obtained by multiplying the number of Kali days elapsed by number of revolutions of planet and dividing by *bhūdina*. For Mercury and Venus, the Sun's mean longitude is the mean longitude.

Mandocca for all the planets except the Moon are generally fixed and given in the texts. For Moon it can be calculated using the same method as for finding the mean planet. *Śīghrocca* for Mercury and Venus is to be calculated like the mean positions of planets. For Jupiter, Saturn and Mars *śīghrocca* is the mean Sun.

It is necessary to point out the innovative method of Nīlakāṇṭha Somayājin (II. 60-79). He has prescribed the corrections, half-*manda*, half-*śīghra*, full-*manda* and full-*śīghra* for Mars, Jupiter and Saturn. For Mercury and Venus, only two corrections, a *manda* correction and *śīghra* correction are required. He has identified the mean position for Mercury and Venus with *śīghrocca* and taken the mean as the *śīghrocca* for the two, as in the case of other planets. This 'breakthrough' in Indian Astronomy which even suggests the heliocentric motion of planets, has not caught the attention of a much later author, Śaṅkaravarman. Though he suggests *Dṛk* system for eclipses, *chāyāgaṇita* etc., he seems to be content with the outmoded *Parahita* system in general. There was a period in Kerala during which *Parahita* was used for *muhurta* etc. and *Dṛk* for eclipses, and things of practical nature. Śaṅkaravarman seems to have emulated this characteristic leading to the dichotomy of astronomical methods.

Different geometrical models of planetary motion are available. In fact the planets are moving round the Sun. For superior planets, the sidereal period is the same whether it is geocentric or heliocentric. Thus the mean longitudes are heliocentric mean longitudes. For Venus and Mercury the revolution of *śīghrocca* is around the Sun and hence the mean *śīghrocca* is same as mean planet around the Sun. When *manda* correction is made, true heliocentric longitudes are obtained; with *śīghra* correction, we get geocentric longitudes. This is the general theory regarding the geometrical model of planetary motion. In fact, the introduction of various circles is only a device for getting the result as observed by Bhāskara I in his comm. on *Āryabhaṭīya* (III. 17):

"tasmādiyam sarvā prakriyā asatyā -yayā grahāṇam spuṭagatiḥ sādhyate |"

This means, "Hence the whole procedure is fictitious, by which the true positions are determined".

But Nīlakaṇṭha Somayājin's model is important, and it helps us to understand the measurement of latitude of planets, required in the computation of combustion. The *mandasphuṭas* of the planets are the heliocentric longitudes and consequently the latitudes are computed from them. Nīlakaṇṭha observes thus (*Tantrasaṅgraha* VII.4b-5a) :

mandasphuṭāt svapātonāt
bhaumādīnām bhujāguṇāt ||
paramakṣepanighnā syāt
kṣepo'ntya śravaṇoddhṛtaḥ |

The latitude is obtained by multiplying the *R* sine of difference in *mandasphuṭa* of planets and the longitudes of their nodes by the maximum latitude and dividing by the *karṇa* at the instant. In symbols, if β is the latitude, ℓ is the maximum latitude, i and n are the *mandasphuṭa* and longitude of the nearer node and k is the *karṇa* then

$$\beta = i \cdot \frac{R\sin(\ell - n)}{k}$$

Example

We shall find the *nirayana* longitude of Saturn when 1864700 Kali days are over. This corresponds to 10-6-2004.

Mean longitude of Saturn	=	$74^0\ 18'$
mandocca	=	236^0
śīghrocca (Mean Sun)	=	$54^0\ 23'$

First Step

mean longitude – *mandocca* = $74^0 18' - 236^0$

= $198^0 18'$ (*Tulādi*)

mandaphala = $+ 138'$

half *mandaphala* = $+ 69' = 1^0 9'$

Making the correction to the mean longitude, we get

$$74^0 18' + 1^0 9' = 75^0 27'$$

Second Step

Corrected mean longitude – *śīghrocca*

= $75^0 27' - 54^0 23'$

= $21^0 04'$

(*Meṣādi* , *Makarādi*)

śīghraphala = $- 121'$

half *śīghraphala* = $- 61' 30'' = - 1^0 1' 30''$

Making the correction, we get

$$75^0 27' - 1^0 1' 30'' = 74^0 25' 30''$$

Third Step

Corrected mean longitude – *mandocca*

= $74^0 25' 30'' - 236^0$

= $198^0 25' 30''$ (*Tulādi*)

mandaphala = $+ 140' = +2^0 20'$

Correcting the original mean longitude with this, we get

$$74^0 18' + 2^0 20' = 76^0 38'$$

Fourth Step

Corrected mean longitude – *śīghrocca*

$= 76^0 38' - 54^0 23' = 22^0 15'$ (*Meṣādi* , *Makarādi*)

śīghraphala $= -127'$

$= -2^0 7'$

Correcting the (last) corrected mean longitude with this we get the true longitude of Saturn $= 76^0\ 38' - 2^0\ 7' = 74^0\ 31'$ This is for Laṅkā at the time of rising of the mean Sun on the day concerned.

THE RATE OF MOTION OF PLANETS

19. The daily mean motion is obtained by multiplying the number of revolutions of the planet in a *caturyuga* by 21,600 and dividing by the number of civil days in a *caturyuga*. For the Sun, add to the mean motion *R* cosine of the *manda kendra* divided by 1550 (*aṇimādya*) or subtract from it according as the *manda kendra* is *Karkyādi* or *Makarādi.* In the case of the Moon add to the mean motion *R* cosine of the *manda kendra* divided by 50 or subtract from it as the case may be.

For the Sun and the Moon the correction is in terms of differentials, $d\left(\frac{r}{R} - \sin m\right) = \frac{r}{R} \cos m \,.\, dm$ if m is in radians. The daily rate of motion of the Sun is given by 59′ 08″ (*dānadharma*).

The correction $= \dfrac{3}{80} \times \dfrac{59'08''}{\frac{180}{\pi} \times 60} \times$ *koṭijyā*

$$= \frac{\textit{koṭijyā}}{1550}$$

Similarly, for the Moon the mean motion is 780′ 35″ (*mṛganilasu*) and

$$\text{the correction} = \frac{7}{80} \times \frac{780'35''}{\frac{180}{\pi} \times 60} \times \textit{koṭijyā}$$

$$= \frac{\textit{koṭijyā}}{50}$$

20. To find the daily rate of motion for other planets, multiply the mean motion by *mandajyā khaṇḍa* (*R* sine difference) and divide by unit of division (*śarāsaśakala*). Add to or subtract from the rate of mean motion according as it is *Karkyādi* or *Makarādi*. Then multiply this by the quantity obtained by subtracting *śīghrajyā khaṇḍa* from rate of *śīghra* motion and divide by the unit of division. Then add to or subtract from the earlier result according as it is *Karkyādi* or *Makarādi*.

THE MEAN SUN AT SOLAR INGRESS

21. Subtract *mandoccca* from the longitude of the Sun at solar ingress ; find *bhujajyā, koṭijyā, bhujaphala* and *koṭiphala* ; note the quadrants *viz.*, *Meṣādi, Tulādi, Makarādi,* and *Karkyādi* ; add to or subtract from *R*, the *koṭiphala* according as it is *Makarādi* or *Karkyādi*, square this, add to the square of the *doḥphala* and find the square root. This is called *vyastakarṇa* (opposite hypotenuse). Multiply *doḥphala*

by *trijyā* and divide by *vyastakarṇa*. Subtract this from or add to the longitude of the Sun at solar ingress according as it is *Meṣādi* or *Tulādi*. The result is the mean longitude of the Sun.

The above stanza gives the method of getting the mean position of the Sun from the true position at a solar ingress. The longitude of the Sun while entering *Meṣa* is 0^0, while entering *Vṛṣabha* is 30^0 and so on. The method prescribed for getting the mean longitude of the Sun corresponding to these is given above. From the longitude, subtract the *mandocca* and find the *bhujajyāphala* $= \frac{3}{80}(R\sin m), \frac{3}{80}(R\cos m)$ and $R + \frac{3}{80}(R\cos m)$, m being the mean anomaly. Let

$$\frac{3}{80}(R\sin m) = A \text{ and } \pm \frac{3}{80}(R\cos m) = B.$$

Then find $\sqrt{A^2+B^2}$ called *sphuṭa koṭi* and $\ell + \frac{3/80(R\sin m)R}{\sqrt{A^2+B^2}}$.

If s is the true longitude of the Sun, then mean longitude = $\ell + \frac{3/80(R\sin m).R}{\sqrt{A^2+B^2}}$. The principle is simple. If s is the mean longitude, and ℓ is the true longitude, then $\ell = s +$ *mandaphala*. Therefore s = $\ell -$ *doḥphala*. The *mandaphala* is normally $3/80\,(R\sin m)$. But greater accuracy is achieved by taking it as $\frac{3/80(R\sin m)R}{\sqrt{A^2+B^2}}$ as is done in the case of *śīghra* correction.

THE SUN'S MOTION IN *RĀŚIS* AND *NAKṢATRAS*

22. At every solar ingress, find the mean longitude (as above), add to the *doḥphala* in the end of the year,

multiply by *bhūdina* and divide by the number of solar days. Then the *māsavākyas* are obtained. Similarly the *vākyas* for *nakṣatras* can be obtained.

The mean longitudes of the Sun at the true solar ingresses are as given below (see III. 6) :

Vṛṣabha	$0^0\ 28'\ 22''$
Mithuna	$1^0\ 29'\ 19''$
Karkaṭaka	$3^0\ 0'\ 27''$
Simha	$4^0\ 1'\ 29''$
Kanyā	$5^0\ 2'\ 4''$
Tulā	$6^0\ 2'\ 5''$
Vṛścika	$8^0\ 1'\ 33''$
Dhanus	$8^0\ 0'\ 38''$
Makara	$8^0\ 29'\ 35''$
Kumbha	$9^0\ 28'\ 37''$
Mīna	$10^0\ 27'\ 59''$
Meṣa	$11^0\ 27'\ 53''$

23. The *vākyas* relating to the years can be obtained by multiplying 365 days 15 *nāḍikās* 31 *vināḍikās* 15 *gurvakṣaras* by 1, 2, 3, etc., . . . By dividing the *vākyas* relating to month, *nakṣatra* etc. by 7 the *saṅkrānti vakyās* etc. from Sundays are obtained.

The method of finding *māsavākyas* were discussed above. If they are divided by 7 the remainders are as follows as days and *nāḍikās* :

Timire	*niratam*	*camare*	*marutaḥ*
2 – 56	6 – 20	2 – 56	6 – 25
surarāt	*ghṛṇibhiḥ*	*javatur*	*dhaṭakaḥ*
2 – 27	4 – 54	6 – 48	1 – 19
nṛvarāt	*sanibhaḥ*	*maṇimān*	*cayakā*
2 – 40	4 – 07	5 – 55	1 – 16

Vṛṣabhāt taraṇerbhavati pragatiḥ

These are the *vākyas* from the month *Vṛṣabha* onwards.

When the true longitude is 360^0 we get the figure $\frac{\frac{3}{80}(R\sin m)R}{\sqrt{A^2+B^2}}$ to be $2^0\,7'$ clearly. Also

$$11^r 27^0 53' + 2^0 7' = 12^r = 360^0$$

as it should be.

$$360 \times \frac{bh\bar{u}dina}{sauradina} = 365.25$$

The quotient is 365. In this way the figure can be obtained for each month. They indicate the number of days elapsed in each month and are called *māsavākyas*, or the statements giving the number of days over the months *Meṣa, Vṛṣabha* etc. The *māsavākyas* are:

kulīna	*rūkṣajña*	*vidhāna*	*matraya*
31	62	94	125
kṣaṇasya	*simhasya*	*suputra*	*catvaraḥ*
156	187	217	246
tathadri	*mināṅga*	*mṛgāṅgi*	*mātulaḥ*
276	305	335	365

In this way using longitudes of the Sun at the ends of *nakṣatras*, (with an interval of $13^0 20'$), 0^0, $13^0 20'$, 26^0, $40'$ . . . 360^0, the *nakṣatravākyas* are obtained.

THE RATIONALE OF *YOGYĀDI VĀKYAS*

24. Find the longitude of the Sun for the solar ingress into *Meṣa* etc. Find the longitude eight days later and continue four times. Find the differences of the longitudes successively. Subtract 8^0 from each. The *yogyādi vākyas* are obtained.

The mean daily motion of the Sun is $59^0 8'$ (*dāna dharma*). At the solar ingress into *Meṣā* the longitude is 0^0. The longitude after 8 days is $8^0 11'$. $8^0 11' - 8^0 = 11'$ (*yogya*). In this way, one can calculate successively. So they are called *yogyādi vākyas.*

We shall find the *vākyas* for the month *Vṛṣabha*. Longitude of the Sun at the time of ingress into *Vṛṣabha* is 30^0.

The longitude after 8 days = $38^0 19'$
The longitude after 16 days= $46^0 40'$
The longitude after 24 days= $54^0 02'$
The longitude after 32 days= $62^0 26'$

Taking the difference and subtracting 8^0, we get

19′ (*dhanyaḥ*) 21 (*putraḥ*) 22 (*kharo*) and 24 (*varaḥ*).

For *yogyādi vākyas* see *Pañcabodha,* III.7.

The first column relates to dates 1-8, the second to dates 9-16, the third to dates 17-24 and the fourth to the dates 25 to the last. The correction has to be applied negatively from the 1st of *Mīna* to 8th *Tulā* and positively from 9th *Tulā* to the end of *Kumbha*. For days less than 8 the correction has to be calculated proportionately. The correction for any date beyond

8 is the sum of the correction for the previous eight days of the month and the portion for the current eight. The proportion for the days beyond 24 has to be done on the basis of the number of remaining days in the month.

The method of finding *viṣudhruva* is not given in the text, though the concept occurs in the verse. The auto-commentary bypasses it. We give following method from *Pañcabodha* (II.12) :

śakābda śāstrārtha vadhāttithiśe
nāptam dinādyam kali nāśa puṇyaiḥ |
gurvakṣarādyam sahitam sunaṣṭam
vivasvataḥ saṅkramaṇa dhruvam syāt ||

Accordingly find the *śaka* year multiply by *śāstrārtha* (725), and divide by *tithiśa* (576). The quotient gives the number of days. Multiply the remainder by 60 and divide by 576. The *nāḍikās* are obtained. Multiply the remainder by 60 and divide by 576 get *vināḍikās* and then continuing, get the *gurvakṣaras*. We can divide it by 7 and use the remainder. One has to add *kalināśapuṇya* (11^{n} 50^{v} 31^{g}) and *saṅkrānti vākya*. The number of days counted from Sunday gives the week day and the other part in *nāḍikās* etc., at the time of solar ingress.

THE CORRECT TIME OF SOLAR INGRESS

25. Find the correction to the mean longitude of the Sun, with *doḥpala, cara* and *deśāntara*. Multiply by 10. To this add the *saṅkramavākya* and *viṣudhruva*. Convert into *nāḍikās, vināḍikās* etc.

When the corrections are made, we get the hour angle traced by the Sun since sunrise and when multiplied by 10, we get the *nāḍikās*, (and *vināḍikās* after conversion). When

viṣudhruva and *saṅkrama - vākyas* are added, the correct time of ingress is obtained. The accurate *saṅkrama vākyas* are from the month *Vṛṣabha* onwards.

	d	*n*	*v*	*gurvakṣara*
lokānām lakṣmaṇāgre	2	55	30	13
vibhāga budha patau	6	19	33	44
dhurvidhau śarma śīghram	2	55	59	49
dhanyā stanyāṅgharaktān	6	24	46	19
jayadhanuṣi kharān	2	27	09	18
lolakhaḍgāmbu śobhāḥ	4	4	32	33
dhasradhitva sibhistaiḥ	6	47	49	24
vigaṇaya hayapān	1	18	15	34
sarva sainyardha gātran	2	39	17	47
gaurān kurvīta norvīm	4	06	41	23
thagaya kṛśanamimam	5	55	11	37
muṣṭīkam bāṇakūṭaiḥ	6	15	31	15

The astronomical computations are made for the mean solar day and the *sāvana* days in the *mahāyuga* are given without reference to their lengths. Thus the positions of planets calculated are for the rising of the mean Sun at Laṅkā. Therefore the correction for the true Sun is effected. For any other place *deśāntara samskāra* or the correction for the longitude are required and the *cara samskāra* or correction for ascensional difference. *Bhujāntara* correction or the equation of time due to the unequal motion of the Sun in the ecliptic is made by the *mandaphala.* After getting these, the time after sunrise is to be obtained. Then arc is multiplied by 10 to get the corresponding units. These four corrections are considered in this work. The *udayāntara* correction or

the reduction to the equator is known only from Śrīpati. Though *Pañcabodha* and *Karaṇapaddhati* etc. do not refer to this, a later work, called *Ganitanirṇaya* (pp. 148-57) incorporates this correction also.

We shall calculate the time of *Vṛṣabha saṅkrānti* in the year *Tāraṇa* Śaka (1926) at Thiruvananthapuram.

Śayana longitude of the

Sun	=	$30^0 + 23^0 54' 52''$
	=	$53^0 54' 52''$
cara	=	$-175'$
doḥpala	=	$1^0 38'$
deśāntara	=	$-0^n 12^v$

The algebraic sum of *cara*, *doḥphala* and *deśāntara* in time units

$$= -24^v\ 30^g$$

Saṅkrānti vākya is $2^d 55^n 30^v 13^g$

The *viṣudhruva* for 1926 is obtained thus when multiplied by 725 and divided by 576. The quotient is 2424. Proceeding with the remainder we get 2424-13-7-19.

d	n	v	g	
2424	13	7	19	+
0	11	50	31	(*kalināśapuṇya*)
2424	24	57	50	

The time of solar ingress is

	d	n	v	g
Saṅkrānti vākya	2	55	30	13

Viṣudhruva	2424	24	57	50
Correction (–)			24	30
	2427	20	7	33

Dividing by 7 we get the remainder to be 5 – 20 – 7 – 33

This took place when this period was elapsed since Sunday. The day indicated by 5 is Thursday. Thus it took place on a Friday at $20^n 7^v 33^g$.

LUNAR MONTHS, SOLAR MONTHS ETC.

26. When the number of revolutions of the Sun in a *caturyuga* is subtracted from the number of revolutions of the Moon in the *caturyuga* the number of *cāndra māsas* in a *caturyuga* is obtained. The number of revolut'ons of the Sun multiplied by 12 gives the number of solar months. The number of *adhimāsas* (intercalary months) is obtained by subtracting the number of solar months from that of lunar months. Multiplying these by 30 we get respectively, the number of lunar days, solar (*saura*) days and the number of days in *adhimāsas*. The number of lunar days decreased by the *bhūdinas* (number of civil days) gives the number of *avama tithis* (days). By adding the *bhūdinas* to the number of revolutions of the Sun, *nākṣatra* days (sidereal days) are obtained.

A few definitions will be helpful. *Sāvana* day is the civil day and is measured by the time between a sunrise and next sunrise. A *tithi* of day is defined as that which ends on the day.

Thus if three *tithis* operate on a day, the first goes uncounted, because it did not end on the previous day. Such a *tithi* is called *avama*.

DIMENSIONS OF THE ORBITS OF PLANETS

27. Multiply the number of revolutions of the Moon by 21600. The *akāśakakṣyā yojanas* are obtained. When this is divided by the number of revolutions of a planet, its *kakṣyā* is obtained ; when it is divided by *bhūdina*, the planet's motion in *yojanas* is obtained. When the Sun's *kakṣyā* is multiplied by 60, the *kakṣyā* of *Aśvini* and other stars is obtained.

The above stanza gives the circumferences of the *ākāśa* and other planetary orbits.

Circumference of the *ākāśa*

= No. of revolutions of the Moon × 21,600

= 55753336 × 21,600 *yojanas*

= 1, 24, 74,72,05,76,000 *yojanas.*

The circumferences of the orbits of various objects are as follows:

Object	*Yojanas*
Sun	28, 87, 666
Moon	2, 16, 000
Mars	54, 31, 291
Mercury	6, 95, 473
Jupiter	3, 45, 50, 133
Venus	17, 76, 421

Saturn	8, 51, 14, 493
Stars	17, 32, 60, 008

The daily motion of planets in *yojanas* = 7906

Yojana is a unit with different definitions. If a *yojana* is taken as 4 miles, the figure 2,16,000 as circumference of the lunar orbit is nearly correct. The other orbits based on this are such that the linear daily motion of each planet is the same.

XVII. To find the diameters of the discs of planets

28. The diameter of the Sun, who is a manifestation of fire is 4410 (*udyadbhāva*), that of the Moon, a manifestation of water is 315 (*śakala*) and that of earth a manifestation of clay is 1050 (*āṭmā nityaḥ*)

29. The visible boundary on a sphere is obtained by multiplying the distance by the diameter of the earth (of the place), adding it to the square of the distance and taking the root. It is also equal to the square root of the square of the sum of the radius and distance reduced by the square of the radius.

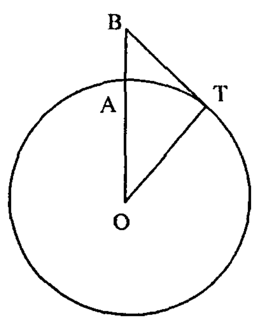


Figure 6.3

Let O be the centre of the sphere and A a place and AB, the height of observer. Let $OA = a$ and $AB = h$. Draw the tangent from B to meet the sphere at T. Then the distance of the visible boundary

$$= BT = \sqrt{OB^2 - OA^2}$$

$$= \sqrt{(a+h)^2 - a^2} \qquad (1)$$

$$= \sqrt{a^2 + 2ah + h^2 - a^2}$$

$$= \sqrt{(2a)h + h^2} \qquad (2)$$

(2) and (1) are stated in the stanza in that order.

30. The diameter of the globe is obtained when the square of the height above the earth is subtracted from the square of the distance of the visible boundary and divided by the height of the observer above the earth.

If the height of the observer $= h$, the distance of the visible boundary $= \sqrt{2ah + h^2}$. Now,

$$\frac{(\sqrt{2ah + h^2})^2 - h^2}{h} = \frac{2ah + h^2 - h^2}{h} = 2a$$

as required.

The author says in his auto-commentary that this gives a practical way of estimating the earth's radius. When one

climbs a tree, wall or the like and measures the apparently flat space up to the visible boundary, the radius of the earth can be estimated.

FINDING THE CIRCUMFERENCE OF THE EARTH

31. Fix two places on the north and south of the equator, and find the distance between them. Then the circumference of the earth (a great circle of earth) is equal to the distance multiplied by 360^0 and divided by the difference in the latitudes (in degrees).

If A and B are any two places on the earth, assumed to be spherical, the method works through. There is no need for taking one in the north and the other in the south, unless a special role for the equator is required. If the earth is spherical, any great circle can be the equator. It appears that the author was aware of the fact that the earth is not exactly spherical.

FINDING THE ORBITS OF THE SUN AND OTHERS

32. Find the *vyastakarṇa* of the Sun and divide *trijyā* by this. This is the *mandakarṇa*. Multiply it by the circumference of the Sun's orbit. This is called *sphuṭa yojana karṇa* which is the radius of the Sun's orbit at the moment.[2]

33. The division by 235 (*māgaṛa*) etc. and also 200 (*jnānīndra*), 420 (*nirūḍhi*) etc. of the *śīghrocca* of Mercury and others have to be used to correct the figure obtained by multiplying the Kali day by the number of revolutions and dividing by *bhūdina*. Do the corrections to *mandocca* of the Moon with three *rāśis* added and to Rāhu after subtracting six *rāśis*.

34. The figures obtained by multiplying the *bhūdina* and dividing by divisors the revised divisors, and their reducibility are mentioned. The desired day of Kali is called *khaṇḍa* and the mean positions on that day of the planets are called *dhruvas*. The Moon's chronograms give the true longitudes of the Moon for intervals of 248 days.

The first part is explained under the next verse. Since the computation of the mean positions of planets involves large numbers, simplified procedure is adopted. A Kali day is fixed and it is called *khaṇḍa*. When *khaṇḍa* is subtracted from the number of Kali days elapsed, we get *khaṇḍa śesa*. Find the mean position for *khaṇḍa śeṣa*. To remove the accumulated error find the mean on the *khaṇḍa* and add or subtract to the mean position as required. These mean positions on the *khaṇḍa* days are called *dhruvas*.

35. Multiply any number by the *bhūdina* and divide by the number of revolutions of a planet. We get a divisor. Multiply this divisor by *bhūdina* and divide by the product of the *ūna śeṣa* or *adhika śeṣa* and the number of minutes in the zodiac (21,600). Then we get the second divisor which is positive or negative.

This requires some explanation. When *bhūdina* is multiplied by any number and divided by the number of revolutions the quotient is called the first divisor. If the remainder is greater than half the number of revolutions add one to the quotient and the remainder can be subtracted from the number of revolutions. Here we get a divisor and *una śeṣa*. Otherwise it is called *adhika śeṣa*. Let

b = *bhūdina*

k = *khaṇḍa śeṣa*

r = number of revolutions

s = number of Sun's revolutions

d = *danādiguṇa kāraka*

m = *mandādi hāraka*

g = *guṇakāra*

h = *hāraka*

The motion during the *khaṇḍa śeṣa*

$$= \frac{k.r}{b} \textit{ revolutions} + k.\frac{d}{m}.\frac{s}{b}$$

The first part of this $= \dfrac{k.r}{b} = k.g.\dfrac{1}{\frac{b.g}{r}}$

where g is any number, called *guṇakāra*. Let us assume that the *ūna śeṣa* is p. It is positive. Then $b.g = r.h - p$. Then $r = \dfrac{b.g+p}{h}$. Therefore,

$$k.\frac{r}{b} = k.g.\frac{1}{b \times g}.\frac{bg+p}{h}$$

$$= \frac{k \times g}{h} + k.\frac{p}{b \times h} \text{ revolutions}$$

$$= \frac{k.g}{h} \text{ revolutions} + k.\frac{21{,}600\, p}{b.h}$$

when the *śakābda* correction is also added to the minutes we get,

$$k \cdot \frac{21{,}600 \cdot p}{b \cdot h} + k \cdot \frac{d}{m} \cdot \frac{s}{b} \cdot = k \frac{21{,}600\, p + \frac{(s \cdot k \cdot d)}{m}}{b \cdot h}$$

$$= \frac{k}{h_2}$$

Here the second divisor h_2 is

$$\frac{bh}{21{,}600\, p + s \cdot \frac{k.d}{m}}$$

Therefore we get the motion in *khaṇḍa śeṣa*

$$= \frac{k \cdot g}{h} \text{ revolutions } + \frac{k}{h_2}$$

Since the computation of the mean longitude involves large numbers, simple procedures are devised. Instead of finding the mean by using the formula mean longitude $= \frac{\text{kali day} \times \text{No. of revolutions}}{\textit{bhūdina}}$ and incorporating the *śakābda* correction, one can use smaller divisors as found above and compute the mean longitude easily.

36. Divide mutually the *guṇa kāra* and *hāraka* till a small remainder is obtained. Place the quotients one below the other and place 1 in the end. Multiply by the third (from the bottom) by the entry below add 1, and drop 1. Continue the process till we are left with two elements.

The method is about forming a *vallī* and the process called *vallyupasamhāra* (see Appendix II)[3] as in the case of solution of linear Diophantine equation.

Let a, b be two positive integers and $b > a$. Divide mutually b by a and continue. We get the quotients q_1, $q_2 \dots q_n$ and remainders $r_1, r_2 \dots r_{n+1}$ satisfying the following.

$$b = q_1 a + r_1$$

$$a = q_2 r_1 + r_2$$

$$r_1 = q_3 r_2 + r_3$$

$$r_{n-1} = q_n r_{n+1} + r_{n+2}$$

Continue till $r_{n+2} = 1$

Consider for example the case when $a = 449$, $b = 12,372$. We get

1	449	12372	27
4	200	249	1
	4	49	12
		1	

The quotients are 27, 1, 1, 4, 12 we shall write this as follows:

Vallī	
27	3031
1	110
1	61
4	49
12	
1	

As explained earlier, we form $4.12 + 1 = 49$

$49.1 + 12 = 61$, $61.1 + 49 = 110$

$110.27 + 61 = 3031$

This is the process of *vallyupasamhāra*

37. When there are two integers, find the Highest Common Factor (*apavartana*) by dividing them till a common factor is obtained, Then mutually divide them, starting with the division of the larger by the smaller. By multiplying the denominators we get a common denominator (of two quotients). Divide mutually *bhūdina* and the difference in the numbers of revolutions of the Moon and *mandocca*. Then the *guṇakāra* and *hārakas* for the Moon's *kendra* (mean longitude – longitude of *mandocca*) can be obtained.

The Sun makes 43,20,000 revolutions in 157,79,17,500 civil days and the number of revolutions per day is

$$\frac{43{,}20{,}000}{157{,}79{,}17{,}500}$$

$$= \frac{576}{2{,}10{,}389} \text{ by } \textit{apavartana.}$$

We can form the *vallī*

365
3
1
6
2
4
2
1

We take a part of this and do the *upasamhāra*

365	$27 \times 365 + 7 = 9862$
3	$3 \times 7 + 6 = 27$
1	$1 \times 6 + 1 = 7$
6	
1	

This is like forming fourth convergent of the continued fraction. (*Yuktibhāṣā,* Appendix *Kuṭṭākāram*, pp. 51-61)

$$\frac{1}{365+}\frac{1}{5+}\frac{1}{1+}\frac{1}{6+}\frac{1}{2+}\frac{1}{4+}\frac{1}{2+}\frac{1}{1}$$

$$=\frac{1}{365+}\frac{1}{3+}\frac{1}{1+}\frac{1}{6+}\frac{9}{20}$$

instead of using $\frac{576}{210389}$ we can use $\frac{27}{9862}$ as an approximation. The error is

$$\frac{576}{210389}-\frac{27}{9862}=\frac{9}{210389\times9862}$$

If we need the mean position of the planet at time T we get

$$\frac{576T}{210389}-\frac{27T}{9862}=\frac{9T}{210389\times9862}$$

The integral part of $\frac{27T}{9862}$ gives the number of revolutions elapsed and the fractional part alone is required to get the mean position. The latter term is generally small and can be ignored or retained. In this way, the successive convergents of the continued fraction corresponding to a *vallī*, which are obtained

by *upasamhāra* can be utilized. For details the reader may refer to *Yuktibhāṣā* (Appendix, p.3).

In this way, by dividing mutually the difference in the revolutions of the mean and its *mandocca* and *bhūdina*, a *valli* can be obtained.

38. Find the Moon's *kendra* (mean longitude – *mandocca*) convert into minutes on a day and add 39^0 17′ (*sukalāmbu*), multiply by the desired *hāraka*, and divide by 21,600. Then, multiply the result by the previous *hāraka*, divide by the *hāraka* and subtract the remainder from *ahargaṇa*. If it is an odd divisor subtract the *adhika śeṣa*. If it is even, subtract the *ūna śeṣa*. This is the *vākya khaṇḍa* for the Moon. The true longitude of the Moon on the *khaṇḍa* date is the *dhruva*.

The reader is supposed to be already acquainted with the method of computing the position of the Moon using *Candravākyas* of Vararuci. We shall explain the theory of this procedure now. The table of Vararuci gives the positions of the Moon for 248 days at sunrise at Laṅkā (the zero position on earth). This starts at the instant when the Moon, *mandocca* of the Moon and *Meṣādi* coincide. The *vākyas* are :

Gīrnaḥ śreyaḥ	:	12^0 03′
Dhenavaḥ Śrīḥ	:	24^0 09′
. . . .	:	. . . .
. . . .	:	. . . .
. . . .	:	. . . .
Bhavet Sukham	:	27^0 44′

At the commencement *Meṣādi* and the Moon are together and therefore the longitude is 0. On the first day the Moon moves by $12^{\circ} 03'$. On the next day the longitude increases to $24^{0} 09'$ and so on. When 248 days are over, the Moon has moved away from *Meṣādi* by $27^{0} 44'$. But nearly 9 anomalistic periods are over. The length of anomalistic period - the time taken by the Moon from *mandocca* to *mandocca* is 27.5545 days. 27.5545 $\times$ 9 = 247.9905 days (nearly 248 days). If this table is used for the next cycle a correction of $27^{0} 44'$ has to be made, because the Moon has moved by that distance. This is called *dhruva.* The figure in the table is called *vākya.* We can use it for any day. But the number of the *vākya* to be used and the correction, the accumulated error, called *dhruva* have to be found out.

Works on Astronomy, *Pañcabodha* (II.2-5), for example give the method of computation. First find the Kali day. Different methods are available for the finding the number of Kali days. After fixing this we shall find the *vākya* to be used. For this suotract 1741650 from Kali days ana divide by 12, 372, 3031 and 248. Note the quotients q_1, q_2, q_3 and let r be the final remainder. It suggests the *vākya* to be used. If the remainder is 192, 192nd *vākya* should be used.

Now we must find the *dhruva* and the accumulated error. For this multiply q_1 by $9^{r} 27^{0} 48' 9'' 44'''$ multiply q_2 by $11^{r} 7^{0} 41' 10'' 16'''$ and q_3 by $0^{r} 27^{0} 43' 28'' 39'''$ and add. Then add $1^{r} 6^{0} 31' 41'' 31'''$. When this figure is added to *vākya* the longitude at sunrise at Laṅkā is obtained. For other places, *deśantara samskāra* and *cara samskāra* are required.

Before proceeding to discuss the rationale, we need some details. We refer to the *vallī* given in the explanation of the v. 37. The successive convergents are:

$$\frac{1}{27}, \frac{1}{28}, \frac{2}{55}, \frac{9}{249}, \frac{110}{3031}, \frac{449}{12372}, \frac{6845}{188611}, \frac{18361}{1332649}, \frac{55209}{1521260}, \frac{766081}{21109029}$$

The *vākyas girnaḥ śreyah* etc., can be derived for the first 248 days. The mean motions of planets, Moon's *mandocca*, and Rāhu are given by the *Pañcabodha* (III.4) :

dāna dharma mṛganīlasutāra
yogarāga śubharam nanu mānī |
dīna tāla nanu rājñi kaviste
pūjyagānagatayo vikalādyāḥ ||

The Moon's mean motion for a day = *mṛganīlasu*

$= 79^0\ 0'\ 35''$

The motion of Moon's *mandocca* in a day = *kaviste*

$= 6'\ 41''$

Mandakendra in one day = $783'\ 54''$

The lunar *jyā* of $783'\ 54'' = 67'$ (*Meṣādi*, negative)

Moon's longitude $= 790'\ 35'' - 67'$

$= 12^0 02'\ 35''$

$= 12^0 03'$, (omitting the seconds).

This can be made accurate using better *Jyā* tables. Mādhava of Saṅgamagrāma gave the tables *śīlam rajñaḥ śriye* ($12^0 02'\ 35''$) etc. The lunar motion undergoes lot of changes and Puliyur Purushothaman Namputhiri, had to change this to, *kṛṣṇaḥ pāyat* ($11^0 51'$). In this way the *vākyas* for 248 days can be found out.

The aim of getting a *khaṇḍa* is to have a day when the Moon is close to *mandocca* and preferably near the (Mean) Sun at Laṅkā when it rises on that day. We can choose a day (*ahargaṇa*) and do as suggested; then find the mean longitude of the Moon, its *mandocca* and the *manda kendra* by subtracting

the *mandocca*. Convert it into minutes. Add 39′ 17″, multiply by 188611; divide by 21,600. Subtract the remainder from *ahargaṇa*. The result is *vākya khaṇḍa* or the day we are searching for. The choice of the division is left to us. One can choose either 12372 or 3031. But one should search for a day which will satisfy our conditions to the maximum extent. In 248 days the Moon moves 6′ 59″ ahead of *mandocca*. When 12 rounds are over that is 248 × 12 days, the motion is 6' 59″ × 12 = 83′ 50″. In 55 days the Moon moves by 83′ 55″. Thus, in 248 × 12 + 55 days, the motion of the Moon is 1′ 43″ ahead of *mandocca*. When this is repeated 4 times, it becomes 6' 52″. In this we can note that the difference fluctuates on either side and in 188611 days the difference is 7′ 1/15. In 248 days the difference is positive and equal to 6′ 59″. In 55 days the difference is negative and equal to 85′ 33″. The difference in 303 days 78^0 33′ and half of this is 39^0 17′ (*sukalāmbu*). This has to be added to the *manda kendra*. An examination of the convergents reveals that the *manda kendra* shou'd be increased by some amount to get accurate results. We are interested only in a *khaṇḍa* which is convenient. It is not far away from the *ahargaṇa* chosen and the *dhruva* also is not large. But it is diffiṣult to decide why the choice fell on 39^0 17′ which is half the movement in 248 + 55 = 303 days, without sufficient research.

Let this be x. Find $\frac{x \times 248}{21{,}600} = k$ when rounded off to an integer. We can assume that the *manda kendra* moves 9 times in 248 days. Thus we have to solve the indeterminate equation $9x - k = 248\,y$. This will lead to the solution. In a problem in a Malalyalam commentary on *Karaṇapaddhati* (Ch.III), the *ahargaṇa* is taken as 1741778. The *manda kendra* together with 39^0 17′ is 13935′ 36″. $\frac{13935'36'' \times 188611}{21600} = 121686$. Instead of using 248 and 9 we take 188611 and 6845 and form the indeterminate equation

$$\frac{6845x - 121686}{188611} = y.$$

One can solve this by using 3031 and 110 instead. Applying the principle given in the text, we can take 3031 as the *hāraka* and get the result which is equal to 1741650, the number used in computation. The rule given in the verse can be interpreted using indeterminate equations. Let $\frac{a}{b} = \frac{p_n}{q_n}$, the n^{th} convergent of the continued fraction. If $\frac{p_{n-1}}{q_{n-1}}$ is the penultimate convergent then $p_n q_{n-1} - p_{n-1} q_n = (-1)^n$. If we have the indeterminate equation (*kuṭṭaka*),

$$bx - ay = c,$$

then $$cbq_{n-1} - caq_n = \pm c$$

From this one can get the solution by the process of *takṣaṇam.*

We shall examine various *dhruvas* used:

(1) The *khaṇḍa* is 1741650 (Kali day) (*amitayavotsuka*).

The mean longitude of the Moon = $1^r 6^0 27' 54'' 19''' 52''''$ (these are respectively called *rāśī, bhāga* (degree), *kalā* (minute), *vikalā* (minute) *tatpara* (1/60 of a second), *pratatpara* (1/3600 of a second).

Mandocca	= $1^r\ 7^0\ 11'\ 5''\ 31'''\ 41''''$
Mandakendra	= $12^r\ 43'\ 10''\ 41'''\ 49''''$
Mandaphala	= $3'\ 46''\ 11'''\ 10''''$ (positive)
The longitude of the Moon	= $1^r\ 6^0\ 31'\ 41''\ 31'''$ (*kaulaṭabhūpāla tanaya*)

(2) 12372 (*rasa gairika*)

Mean Moon	$= 9^r\ 27^0\ 18'\ 10''\ 19.73'''$
Mandocca	$= 9^r\ 27^0\ 18'\ 9''\ 30.85'''$
Manda kendra	$= 0^r\ 0^0\ 0'\ 0''\ 48.88'''$
Mandaphala	$= 0^r\ 0^0\ 0'\ 0''\ 35.78'''$ (negative)
Longitude of the Moon	$= 9^r\ 27^0\ 18'\ 9''\ 43.95'''$ (corrected to $44'''$) (*vividham nijavasarodham*)

(3) 3031 (*kulīnāṅga*)

Mean Moon	$= 11^r\ 7^0\ 31'\ 1''\ 15.02'''$
Mandocca	$= 11^r\ 7^0\ 32'\ 44''\ 20.00'''$
Mandakendra	$= 12^r\ (1'\ 13''\ 4.98''')$
Mandaphala	$= 0^r\ 0^0\ 0'\ 9'''\ 1.8'''$ (positive)
Longitude of the Moon	$= 11^r\ 7^0\ 31'\ 10''\ 16.20'''$ (*tape nohyam kulāsanaipuṇyam*)

(4) 248 (*devendra*)

Mean Moon	$= 0^r\ 27^0\ 44'\ 5''\ 19.67'''$
Mandocca	$= 0^r\ 27^0\ 37'\ 6''\ 10.85'''$
Mandakendra	$= 0^r\ 0^0\ 6'\ 59''\ 8.82'''$
Mandaphala (negative)	$= 0^r\ 0^0\ 0'\ 36''\ 40.62'''$
Longitude of the Moon	$= 0^r\ 27^0\ 43'\ 28''\ 39.05'''$ (corrected to 39) (*dhiga hara laghu satroṇam*)

These details supply the rationale of using these figures in computation of *dhruva*.

39. In the procedure for finding the *dhruva*, the *guṇakāras* and *hārakas* are obtained by mutual division. They

can be positive or negative. The difference between the Moon and *mandocca* on the *khaṇḍa* day has to be multiplied by a divisor and divided by the divisor above (in the list of divisors) and the remainder has to be subtracted from the *ahargaṇa.*

This practically reiterates what has been said earlier. But there is lack of clarity in the verse.

40, 41.

Multiply the numbers of intercalary months in the *mahāyuga* by the number of Kali years elapsed, divide by the number of Sun's revolutions (multiplied by 85), multiply by *bhūdina* (corrected to lunar units) and subtract from it the Moon's *kalyādidhruva*, multiply by *bhūdina* and divide by 360. This gives the Kali day for finding the intercalary months. The divisors here are got by mutually dividing the number of intercalary months in the *mahāyuga* and *bhūdina.*

We are finding the number of intercalary months over the time concerned. This is equal to

$$\frac{\text{no. of intercalary months in the } mah\bar{a}yuga \times \text{Kali years over}}{\text{no. of revolutions of the Sun}}$$

$$= n \text{ (say)}$$

But this is not generated from the beginning of the Kali *yuga*. The dhurva of the Moon at the beginning of Kali was $6^0\text{–}23'\text{–}36''\text{–}42'''\text{–}11''''= d$ (say). The number of intercalary months generated by this is $\frac{d}{360}$. Therefore number of intercalary months is $n - \frac{d}{360}$. The number of days required to generate this is :

$$\left(n - \frac{d}{360}\right) \times \frac{bhudhina}{\text{no. of intercalary months in the epoch}}$$

This gives the *khaṇḍa.*

However, if it is required to calculate fresh periods, with the passage of time, form the *vallī* got by mutually dividing number of intercalary months and *bhūdina.*

$$bh\bar{u}dina = 1577917500$$

$$\text{No. of intercalary months} = 1593336$$

In one *mahāyuga* exact numbers of Moon's revolutions are not over. By *Śakābda samskāra* it gets reduced by $\frac{9 \times 200}{85}$. Therefore we take 85 × *bhūdina* and 85 × revolution of the Sun for calculation. When it is done we get the required continued fraction to be

$$\frac{135431760}{134122987500} = \frac{1}{90+}\frac{1}{2+}\frac{1}{1+}\frac{1}{36+}\frac{1}{1+}\frac{1}{5+}\cdots$$

The convergents are

$$\frac{1}{990}, \frac{2}{1981}, \frac{3}{2971}, \frac{110}{108937}, \ldots$$

42. Intercalary months can occur between the mean positions, and true positions, between the new Moon days and between solar ingresses. Thus they are of four kinds.

The length of a lunar month is 29½ days and the lunar year is shorter than the solar year by about 10 or 11 days. The *cāndra* month starts on the *pratipat* of the bright half (or the end of New Moon) and ends by the end of New Moon.

The month containing the solar ingress into *Meṣa* is called *Caitra*, that containing the solar ingress into *Vṛṣabha* is called *Vaiśākha* and so on. Because of the difference in the lengths of solar and lunar years, an excess of a lunar month accrues every three solar years. Thus there may be a month without solar ingress. This is called *adhimāsa*. This may be with reference to the mean positions of the Moon or true positions. They are called respectively *madhyādhimāsa* and *sphuṭādhimāsa*.

It may also happen that two solar ingresses occur in the same lunar month, though very rarely. Then it s called *amhaspati*. The previous or later month will be an *adhimasa* (without solar ingress). This is called *samsarpa*. *Samsarpa* and *amhaspati* occur always together. Thus there are two kinds of solar months, with mean or true positions and two kinds of lunar months, with true or mean positions, constituting four kinds of intercalary months.

THE *DṚK* SYSTEM OF 4708 KALI

43. I shall now give the system of astronomy tallying with observation, which was enunciated in the Kali year 4708 (*janasabhā*) after finding from observations, the differences in the *Parahita* system.

The author now gives the *Dṛk* system which was introduced to rectify the errors in the *Parahita* system.

44. The number of revolutions of the Sun etc. are given by

4320000	(*jnānanighna phalavit*)
57753320	(*narāṅga guṇa satsumam*)
488122	(*rurupadārjavam*)
2296863	(*lakṣadatṛdhararāṭ*)

17937100	(*anekasugaḷotsukaḥ*)
364166	(*kṣitipabhūtalam*)
7022272	(*śrīsakhī khuranasā*)
1577917500	(*anīśasāyudhasusamśayaḥ*)

The number of revolutions is in the order: the Sun, Moon's apogee, Mars, Mercury, Jupiter, Venus, Saturn and Rāhu. Nilakaṇṭha Somayājin settles these after experimentation, and gives slightly different figures in his *Siddhāntadarpaṇa* (vv. 2-5).

45. The *Kalyādidhruva* (zero positions at Kali) for Sun, Moon, Moon's *mandocca*, increased by 90^0, Moon's *pāta* (Rāhu) decreased by 180^0, Mars, Mercury, Jupiter Venus and Saturn are:

ājñatatpara	(621600)
hṛtam	(68)
harihayāsannasya	(1071828)
bhimarbhakaiḥ	(1454)
malāśobhi	(4535)
jalārthi	(738)
ratnaṇṛpa	(1002)
daityarīḍya	(1218)
nārīstanaiḥ	(620)

where the figure is subtractive for Mars, Mercury and Saturn and additive for others.

No clue is available for interpretation. Several authors have given several figures. These are supposed to be the figures in the *Dṛk* system. The figures if taken as *rāśi*, degrees, minutes, seconds etc. do not seem to tally with any system.

46. The *mandocca* of the Sun is $2^r 18^0 14'$ (*bhatyudayādriḥ*). Those of Mars, Mercury, Jupiter, Venus and Saturn are

4^r 08^0 $33'$	(*budbudanābhaḥ*)
7^r 01^0 $47'$	(*saṅghaṭanārtham*)
5^r 22^0 $05'$	(*munirudrāmśaḥ*)
2^r 21^0 $40'$	(*nirbhayarāṣṭre*)
8^r 02^0 $09'$	(*dhenuranandat*)

Except for the Moon, the *mandoccas* of planets are generally fixed. According to *Pañcabodha* (IV.2), the *mandoccas* in *Dṛk* are

2^r 18^0 $14'$	(*vandyo jayaśrīḥ*)
4^r 7^0 $33'$	(*balasūnu bhānuḥ*)
7^r 7^0 $47'$	(*sarvārthānātha*)
5^r 22^0 $05'$	(*munīndrarāmaḥ*)
2^r 21^0 $40'$	(*abhīṣṭartrau*)
8^r 02^0 $09'$	(*dhanaratnadāram*)

Nīlakaṇṭha Somayājin's *Siddhāntadarpaṇa* (p.45) gives slightly different figures. The reading *saṅghajanārtham* which occurs in II.32 in this work means 7^r 8^0 $47'$; this is perhaps more accurate.

47. The radii of Mars and others are given by multiplying

yajamānanāsa	(700581)
puṭīdhamat	(5911)
kīṭa	(11)
sunāḍikā	(1307)
andhajaḥ	(890)

by 10, and dividing by

trijagat	(382)
nadīna	(80)
vistāra	(264)
māyā	(15)
kalabha	(431) and adding 5.

There are many points to be noted. They appear to be radii of the epicycles of Mars and others in the *Dṛk* system. Usually *paridhis* or circumferences are given. Moreover the values at the ends of odd and even quadrants are also given in *Āryabhaṭa* system as in v. 9-10 of this chapter. Even if the author wanted to give revised figures, a straightforward way as in the earlier stanzas could have been adopted. But, why does he resort to a cumbersome method and make the contents unintelligible? Do they mean any thing else? One needs to investigate thoroughly before drawing a conclusion.

48. The *sīghra* diameters of Mars and others are obtained by multiplying

upameyarāja	(821510)
mantrī	(25)
dyunirmita	(6501)
tamolaya	(1356)
nāḷadeha	(8390)

by 10 and dividing by

āḍhya	(40)
videśagulikā	(193584)
dhvanikṛt	(104)
cikitsā	(716)
yoga	(31)

All the remarks on stanza 47 apply to this also.

49. The *mandajyā, śīghrajyā* and *koṭis* can be got by dividing the arc into hundreds and thousands and the *mandaphala* and *śīghraphala* have to be obtained with the respective radii. Others are as in *Parahita* system.

In the *Āryabhaṭa* system the *R* sine chord is given by dividing the arc of 90^0 into $3^0\,45'$ each and for the arcs in between, it is obtained by interpolation. More accurate methods using the principle $d(\sin\theta) = \cos\theta\, d\theta$ is given in Chapter IV or even the infinite series is available. One can divide the angle 90^0 in 100 or 1000 units and find out the values of *jyās.*

50. Find the *koṭīphala* with the sign according to *Karkyādi* or *Makarādi,* add it to or subtract from *R* accordingly, square it and add it to the square of the *manda phala,* and find the square root. The *manda karṇa* is obtained by dividing the square of *trijyā* by this. Multiply this by the radius of the *kakṣyā* and divide by 21,600 to get the *sphuṭa karṇa.* This is the case with Mars and other planets. For the Moon it is like that of the Sun.

Some words are wanting in the verse. The translation is given in accordance with standard definition. The idea of squaring *manda phala* is not referred to. Nor is the extraction of the root. Moreover dividing by 21,600 is not given for the second part. One can change the first two lines of the verse thus:

kheṭasya sphuṭakoṭimandaphalayorvargaikyamūlam tatha |
stena syād vihṛtārdhavistṛtikṛtirmandaśrutistad guṇāt ||

According to standard texts

$$vyasta\ karṇa = \sqrt{\left(R + \frac{a}{b} R \cos m\right)^2 + \left(\frac{a}{b} R \sin m\right)^2}$$

where a and b are the radii of *mandavṛtta* and *kakṣyavṛttas* respectively, R is *trijyā* and m is *mandakendra*, and '±' sign is used according as *mandakendra* is *Makarādi* or *Karkyādi.*

$$Mandakarṇa = \frac{R^2}{vyasta\ karṇa}$$

$$Sphuṭayojanakarṇa = \frac{Mandakarṇa \times \text{Radius of } kakṣyāvṛtta}{21{,}600}$$

51. For Mars and others, divide the Moon's diameter in *yojana* by 16, 12, 10, 8 and 14 respectively, multiply by *trijyā* and divide by *sphuṭayojana karṇa.* For the Sun and the Moon also the same method is followed.

52. Multiply the diameter of the earth by the Sun's *sphuṭa yojana karṇa* and divide by the difference in the diameters of the Sun and the earth. This gives the shadow cone which is on one-side of the earth. Find the difference of this from the Moon's *sphuṭayojana karṇa*, multiplied by the earth's diameter, and divide by the product of the radius and the *sphuṭayojana karṇa* of the Moon to get the shadow in minutes.

53,54. Mutually divide the figure obtained by multiplying by 2 the sum of the numbers of revolutions of the Sun and Rāhu and the number of lunar months in a *mahāyuga* and form the *vallī* and find the divisors. Multiply these divisors by the *bhūdina* and we get the *hārakas* for eclipses. Find the mean positions of the Sun and the Moon and Rāhu, subtract the longitude of the Rāhu from the mean Sun and convert in to minutes. Multiply it by 3803 (lunar days), divide by 21,600 (*anantapura*) multiply the quotient by 716 (*tarkārtha*) divide by 3803 (*lunadaga*) and multiply the remainder by *bhūdina*, divide by the number of lunar months and subtract from the number of kali days to get the *grahaṇa khaṇḍa* of the Sun and the Moon.

This is for solar eclipses. For lunar eclipses, longitude of Rāhu has to be subtracted from the mean longitude of the Moon.

The number of Sun's revolution is 43,20,000. *Bhūdina* = 1577947500. The number of revolutions of the Moon = 57753320. The number of revolutions of Rāhu = 232300.

2 × difference of number of revolutions of Rāhu and the Sun = 9104600

No. of lunar months = 53433320

The required continued fraction is

$$\frac{9104600}{53433320} = \frac{1}{5+}\frac{1}{1+}\frac{1}{6+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\cdots$$

METHOD OF REFORMING THE COMPUTATIONAL METHODS AT A DESIRED TIME

55. Find the difference between the observed position and computed position of the planet into minutes and multiply by the *bhūdina*. Multiply the number of revolutions by 2160274261 and divide by the earlier figure. It has to be added to or subtracted from the result according as it is less or more. In the case of the Sun the procedure is different. One has to verify the Kali days, mean position, *khaṇḍa* and *dhruva*.

CONCLUSION

56. The Kali year 4921 in the 28th *mahāyuga* in the *kalpa* of Vaivasvata Manu, who is the seventh of the fourteen Manus with seventy *mahāyugas* each, since creation and after the deluge, is now in force.

57. In this work of mine which indicates the knowledge of the Earth, Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn and the stars, let the wise people interested in acquiring knowledge in astronomy, show excitement.

The interesting aspect of this verse is the reference to earth among other planets. In Indian astronomy, earth has no specific role except that computation is done with reference to earth as geo-static system is generally followed. The notion is the rotation of *bhacakra* or circle of asterisms.

58. Oh! The resident of *Lokamalayārkāvu*! Mother of the worlds, who is capable of being worshipped by divine beings, shower the auspicious things to the

(work) *Sadratnamālā*, which adorns thy feet that have the radiance of the verses of the group of divine beings and sages, which is made of gold, and which gives pleasure to the world, and to those who wear it in their necks.

This is capable of many interpretations. The work is called *Sadratnamālā*, a garland of good gems. The term '*tridaśa muni*' *susaṅghātapadyoccabhāḍhya*' an adjective of the term *sadratnamālā* means "having the radiance of the loud hymns of the divine beings and sages". The work which is *Sadratnamālā* is placed at the feet of the Goddess, and the feet are worshipped by divine beings and sages, and *Sadratnamālā* also gets radiance of these beings. Also *ghāta* means product and '*tridaśa muni susaṅghāta*' means the well formed product of 3, 10 and 7 which is 210. It indicates roughly the number of verses namely 3×10×7 =210 ; '*bhāḍhya*' can mean that this is to be increased by *bha*, which is 4 or 27 Thus total number may be 214 or 237.[4]

However it indicates roughly the number of verses. '*Svarṇamaya guṇa yuta*' means the garland endowed with a golden cord (which unites the gems). As a work on astronomy it means with *guṇa* (*R* sine) which is *svarṇamaya* (positive or negative). The term '*Lokāmbe*' is not allowed in non-Vedic Sanskrit, *vide – ambārthanadyorhrasvaḥ* (*Aṣṭādhyāyī* VII. 3. 107). It should be '*lokāmba*'[5].

The work ends with the word *sanmaṅgalāṇi*, to indicate the wish for auspicious occurrences.

NOTES

1. S. Madhavan, "Quasi-Keplerian Model of Sūryasiddhānta", *Tantra Saṅgraha*, Indian Institute of Advanced Study, Simla, 2002.
2. The auto-commentary ends here. The remaining stanzas are also translated and explained below.

3. *Cf.* Kuṭṭakavivaraṇam, *Bījagaṇita* of Bhāskara II.

4. See Appendix of *A Modern Introduction to Ancient Indian Mathematics*, T. S. Bhanamurthy, Wiley Eastern Limited, 1992.

5. The *Vārttika, Chandasi vā iti vaktavyam* suggests it can be used in Vedic language. Also, the author might have expressed his deep devotion by imitating the Malayalam word '*Amme*'. Words in local language or their variants can some times be used. Śrīharṣa uses the word *iṅgāla*, in the language of Kānyakubja in his *Naiṣadhīyacarita* (I. 9):

   '*vitenuriṅgālamivāyaśaḥ pare*'

   The word *iṅgāla* means 'burnt log of wood'.

   Also '*Lokamba siddhasevye*' appears to indicate the Kali day on which the work was completed. By assignment of *Kaṭapayādi* numerals it is indicates 1797313. This corresponds to 10th December 1819.

APPENDICES

TECHNICAL TERMS

TABLES

APPENDIX I

ELEMENTS OF ASTRONOMY

Who is not enchanted by the myriads of twinkling stars that adorn the dark blue velvet of the night sky? The advancing night which sprinkles stars all over the sky excites the poet's imagination who lets his fancy roam everywhere. The average man, however is forced to be indifferent to the celestial phenomenon. He finds pleasure in slumber after toiling and moiling during the day. But there is one man who is really serious, trying to learn about the stars. He is the star-gazer. He searches for his celestial companions with the guidance of his telescope and silently engages in computation. But, what is he actually doing? What does he measure and what does he compute? What is the frame work in which he does his operations? One needs to know these before taking to a serious study of Astronomy.

The first task is to identify the stars. The primitive man to whom the stars were pieces of wonder imagined fanciful stories about them. An old Malayan story asserts that the stars were the children of the Moon - mother who brought her children out only during the night, when jealous Sun who had no children, was not present. The Milky Way used to be identified with the celestial Ganges. Despite these descriptions of excited imagination, the early man took great efforts to study the stars. The method of identification of stars is much like identifying a house in a city; give the name of the street and the number of the house. Since it is difficult to identify the stars unless they are sufficiently bright, stars are arranged in groups called constellations first and then with the constellations, the stars are identified. The constellations are given names after the animals or objects which they are supposed to resemble. Sometimes they are named after characters in mythology.

'*Saptarṣimaṇḍalam*' for instance is named after the seven sages, Marici, Vasistha, Angiras, Atri, Pulastya, Pulaha, and Kratu. The faint companion of Vasistha is named after Arundhati.

The general method followed in the West is to give a Latin name to a constellation and name the individual star as Alpha, Beta, Gamma etc. of the constellation in descending order of brightness. Thus Canis major is a group of stars supposed to represent the figure in the form of a dog. The brightest star in the group is called Alpha Canis Majoris. This star is also known as Sirius (*Lubdhaka* in Sanskrit) and is the brightest star in the sky. Bright stars have generally individual names. There is an important group of constellations called the Zodiacal constellations viz., Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius and Pisces. The constellation Aries is so called because of its supposed resemblance to a ram. The brightest star in the constellation is Alpha Arietis; the second is Beta Arietis and so on. Indian astronomy refers to twelve rasis or signs as *Meṣa, Vṛṣabha, Mithuna, Karkaṭaka, Siṃha, Kanyā, Tulā, Vṛścika, Dhanus, Makara, Kumbha* and *Mīna,* which are supposed to be same as above. But *Meṣa* represents a portion of Zodiac of length 30^0 *Vṛṣabha* represents the region of length 30^0 that follows, and finally *Mīna* represents the last 30^0 of the Zodiac. One striking feature of Indian astronomy is that though *Meṣa* represents a zodiacal *rāśi,* no constellation or group of stars is identified as *Meṣa.* Similarly none of the 12 Zodiacal rasis is represented by an actual group of stars identified as *Meṣa.* On the other hand, *Meṣa* is identified with *Aśvinī, Bharaṇī* and the first quarter of *Kṛttikā. Vṛṣabha* is identified with 2nd, 3rd and 4th quarters of *Kṛttikā, Rohiṇī* and first two quarters of *Mṛgaśīrṣa* and so on. Thus all the 27 *nakṣatras* are distributed among the 12 zodiacal *rāśis,* each sign receiving 2 *nakṣatras* and a quarter. These *nakṣatras* are actually, represented by groups of stars. For example, *Aśvinī* is a constellation of three stars resembling the

face of a horse. Each *nakṣatra* has a principal star or *yogatāra*, which is generally a bright star of the group. However the two classifications do not completely coincide. *Śravaṇa* or Altair, the principal star of the *nakṣatra śravaṇa* is in *Makara rāśi.* One may expect it to be in the constellation Capricornus. But it is actually a star in the constellation Aquila. It is true that there are common stars in the two classifications. But they are not completely identical. But it is not important as the *Meṣa rāśi,* and the stars *Aśvinī, Bharaṇī* and the first quarter of *Kṛttikā* are identical and similarly for the other zodiacal *rāśis* and the corresponding *nakṣatras.* We have given illustrations of the constellation Leo, which consists of the principal stars of *Magha* (Regulus), *Pūrvaphalgunī* (Delta Leonis) and *Uttaraphalgunī* (Denebola) and Leonis and Dipper, the constellation. (See figures 1 and 2).

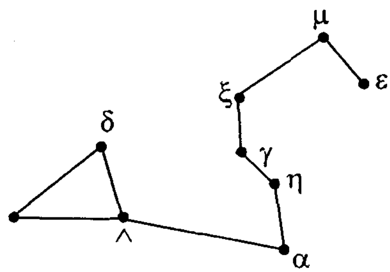


Figure 1.

The Dipper (*Saptarṣimaṇḍalam*)

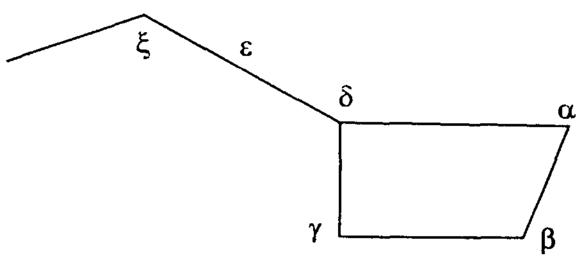


Figure 2.

We shall give below principal stars (*yogatara*) of the 27 *nakṣatras*, and the corresponding names in the West,

Nakṣatra	Western Name
Aśvinī	Beta Arietis
Bharaṇī	41 Arietis
Kṛttikā	Eta Tauri (Alcyon)
Rohiṇī	Alpha Tauri (Aldeberran)
Mṛgaśīrṣa	Lambda Orionis
Ārdrā	Alpha Orionis (Betalguese)
Punarvasu	Beta Geminorum (Pollux)
Puṣya	Delta Cancri
Āśleṣa	Alpha Cancri
Magha	Alpha Leonis (Regulus)
Pūrvaphalgunī	Delta Leonis
Uttaraphalgunī	Beta Leonis (Denebola)
Hasta	Delta Corvi
Citra	Alpha Virginis (Spica)
Svātī	Alpha Bootis (Arcturus)
Viśākhā	lota Lobarae
Anurādhā	Delta Scorpii (Scorpiouis)
Jyeṣṭhā	Alpha Scorpii (Antares)
Mūla	Lambda Scorpii
Pūrvāṣaḍhā	Delta Sagittarii
Uttarāṣaḍhā	Delta Sagittarii (Sigma)
Śravaṇa	Alpha Aquilae (Altair)
Śravishṭa	Alpha Delphini
Satabhiṣak	Lambda Aquarii
Pūrvabhādrapada	Alpha Pegasi
Uttarabhādrapada	Alpha Andromedae
Revatī	Zeta Piscium

In the book *Popular Hindu Astronomy* by Kalinath Mukherji an attempt has been made to identify *Meṣa, Vṛṣabha*

etc., i.e., zodiacal *rāśis* as constellations. The identification is done with the help of scriptures and Sanskrit Literature. However there is no conclusive proof to show that such a system of describing the *rāśis* as constellations was actually in force in ancient India. It is likely that such a system was prevalent in ancient India and the knowledge of this system was lost with the passage of time.

One understands what is meant by a sphere. Any section of a sphere by a plane is a circle. When the plane passes through the centre of the sphere, the section is called a great circle, otherwise a small circle. In the strict sense of the term, the earth is not spherical in shape, but spheroidal. But as an approximation we shall treat the earth as a sphere and build our concepts. Any observer of the sky notes that the celestial bodies rise, move upwards, and set. Dynamical considerations force us to conclude that the earth rotates about an axis. We observe that the rotation is from west to east. This axis meets the earth in two points on earth called the North and South Poles. The terms *Meru* and *Baḍavāmukha* are used to designate these in Indian astronomical literature. All the points equidistant from the two poles lie on a great circle called the equator, known as *Nirakṣarekha* in the Indian system. All circles that are passing through the North and the South poles are called meridians or circles of longitude and small circles along planes parallel to that of the equator are called parallels of latitude. The meridian which passes through Greenwich is called the Universal Meridian. For any place on earth, the terrestrial longitude is determined thus. Draw the circle of longitude through the place A. Let the Greenwich meridian and meridian through A meet the equator at G' and A' respectively. Then the length $G'A'$ which is the same as the angle subtended by $G'A'$ at the centre of the earth is the longitude. Longitude varies from 0 to 180^0 and can be east or west of Greenwich. The parallels of latitude

determine the position of a place north or south of the equator. For the place described above, terrestrial latitude is North or South according as the place is north or south of the equator. The terrestrial latitude is measured by the arc of the meridian through A intercepted between A and the equator. It varies from 0 to 90^0. For an observer in the North Pole there will be no north, east or west. There is only South. Similarly for an observer in the South Pole, there is only North.

For any observer, the sky appears in the form of a hemispherical dome with the stars as points of light spread on its surface. Naturally an astronomer imagines a celestial sphere around him. He, treated as a point, is at the centre of the sphere. He finds the positions of the stars and other celestial bodies as seen on the sphere. The star Sirius, for example is at a distance of 8.7 light years from the earth, and the star Alpha Centauri is at a distance of 4.3 light years. But he uses the projections of these on the celestial sphere for his immediate study, through the actual positions are required in some other contexts. Now the properties of the sphere and methods of spherical Geometry and Trigonometry can be effectively applied to study the movement of celestial bodies.

Given any two points, we can always draw a great circle joining them. The distance between any two points on a sphere is measured by the arc of great circle joining them. This is taken to be the angle subtended at the observer's position by the two points. The term horizon is used in common life and one is intuitively aware of what it is. Varahamihira the celebrated astronomer of Ujjain, defines it as a circle along which the sky and the earth appear to meet. More formally, we can define horizon as the great circle of the celestial sphere intercepted by the tangent plane at the earth's surface at the observer's position. The point of celestial sphere that is vertically overhead is called the Zenith (*Z*) and its antipodal

point, the point diametrically opposite to it, is called the Nadir (*N*). The earth's polar axis when extended in either direction meets the celestial sphere in the North and South celestial poles. The North Pole is conveniently located with the help of the pole star. The point on the horizon below the North Pole is called the North Point. From this the South, the East and the West points can be fixed. One important principle is that the height of the pole above the horizon is equal to the latitude of the place. For any circle on a sphere, the diameter of the sphere perpendicular to the plane of the circle is called its axis and the points of intersection of the axis with the sphere are called poles. For any circle, a great circle through its poles is called a secondary. The great circle with the north celestial pole and south celestial pole for poles is called the celestial equator. The celestial equator divides the celestial sphere into two hemispheres, the northern and southern hemispheres containing respectively the north and south celestial poles. The meridian of a place is the great circle passing through the Zenith, the Nadir and the poles. Verticals are secondary to the horizon and the vertical through the East and West points are called the prime vertical. Any careful observer can see that the Sun is having an eastward motion in the sky with respect to the fixed stars. Observe the eastern horizon before sunrise. Certain stars will be visible near the horizon which gradually pale into insignificance with the arrival of the Sun at the eastern horizon. Repeat the process continuously for a few days. One can notice the group of stars visible near the horizon changes continuously suggesting thereby an apparent motion of the Sun eastwards with respect to the stars. The Sun completes the revolution with respect to the stars in the course of a period called a sidereal year. The apparent path of the Sun is called the ecliptic. This apparent motion is actually due to the earth's revolution, in the orbit

around the Sun. The Zodiac or Zodiacal belt consisting of the constellations Aries, Taurus, Pisces . . . covers the ecliptic. The ecliptic is defined as a great circle of the celestial sphere and it is inclined at about 23^0 27′ to the equator. The points of intersection of the celestial equator and the ecliptic are called the first point of Aries (γ) and the first point of Libra (Ω̲). The point at which the Sun leaves the southern hemisphere to the northern hemisphere in the west to east motion along the ecliptic is called the First point of Aries(γ). The other point is called the First point of Libra (Ω̲).

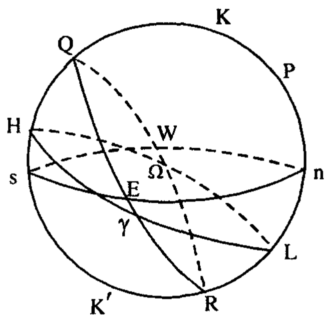


Figure 3.

E	- the East point.
W	- the West point.
P	- the North celestial pole
ns	- the celestial horizon.
QR	- the celestial equator.
HL	- the ecliptic.
γ	- the First point of Aries.
Ω̲	- the First point of Libra.
K and K′	- the poles of the ecliptic.

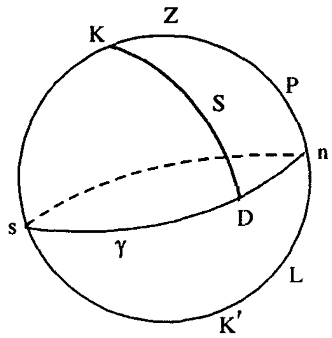


Figure 4.

HL	- the ecliptic
K and K'	- the poles of the ecleptic.
S	- a celestial body.
D	- the foot of the secondary through S.
γD	- the (celestial) longitude of S.
SD	- the (celestial) latitude of S.

It is necessary to acquaint oneself with the system of co-ordinates used for fixing the positions of stars and planets. Since we are not using all of them in the book, we shall discuss some of them which are relevant to the understanding of the concepts in Astronomy. The height of any body above the horizon is measured by its angular distance above the horizon along the secondary to the horizon through the body, and is called the altitude. If *S* is a body and *SD* is a secondary to the horizon *D* being the foot of the secondary then *SD* is the altitude. The angular distance measured from the North point (or the South) eastwards upto *D* is called the azimuth of the body. In this system the coordinates are with reference to the horizon. We can choose equator and a suitable origin for defining another system. Let *S* be a body. Draw *SD* secondary to the equator. *D* being the foot of the secondary. Then γ*D* measured eastwards

is called the right Ascension and SD is called the declination. The declination is measured positive or negative depending on the hemisphere to which S (North or South) belongs. This system is generally used to give the positions of stars. We now start with the ecliptic and define another system. Let S be a body and SD, secondary to the ecliptic. γD measured eastwards is called the celestial longitude and SD is called celestial latitude. SD is measured north or positive and south or negative as the case may be. See figure 4 for a representation.

For any body, $\angle ZPS$ measured westwards is called West hour angle and measured eastwards is called East hour angle.

In Indian system also the similar concepts are used. The pole star is called *Dhruvanakṣatra* and the pole is called *Dhruva*. The equator is called *Viṣuvadvṛtta* and the ecliptic is called *Apamaṇḍala*, *Ravimārga* or *Krāntivṛtta*. Longitude is known as *Sphuṭa* and the latitude is known by the name *Vikṣepa*. *Krānti* is the term used for declination. Altitude is called *nata* and the zenith distance is known by the term *unnata*. Though right ascension is not generally given any name, the concept is used. One can call it *Viṣuvadvṛttīyasphuṭa*. *Kalāmśa* is the term used to refer to the hour angle. There are many other equivalents, which are explained then and there in the text and in the list of technical terms. The principal meridian was taken to pass through Ujjain and the point of intersection of this with equator is called Laṇkā.

There is a need to explain *ayanacalana* or precision of the equinoxes. The first point of Aries is having a retrograde motion in relation to the stars at the rate of about 50″.2 per annum. In Indian system the longitudes are sidereal. In others they are measured from a fixed point of the ecliptic (w.r.t. stars) called *Meṣādi*. Longitudes measured from *Meṣādi* are called *nirayana* longitudes and those measured from first point of

Aries are called *Sāyana* longituides. The two differ by a quantity called *ayanāmśa* the value of which is a disputed one.

Due to the rotation of the earth stars and celestial bodies move in a small circle parallel to the equator. It is called *ahorātravṛtta* and its radius is called *dyujyā*. The term *trijyā* is used for the radius used in R sine tables and is taken to be 3438′ in the Āryabhata school.

The notations used are explained at the time of the first occurence in the text. Figure 1 gives the normal motions used. The latitude of a place is denoted by φ , the Sun's longitude by ℓ , declination by δ, obliquity by ω (called *paramakrānti*).

APPENDIX II

KUṬṬĀKĀRAGAṆITA

The development of Astronomy and Mathematics in Kerala during the period from 4th Century A.D. to 19th Century A.D. had been very significant. Starting from *Cāndravākyas* of Vararuci (4th Century A.D.) to *Sadratnamālā* (1823 A.D.) there are a number of works of great intrinsic importance. The mathematical devices used in these works include computational devices, shortcut methods to circumvent brutal force methods involving labour and time, Spherical Geometry and Trigonometry, pioneering methods in Calculus, successive approximation, inverse interpolation and a host of many other things which still remain unexplored without seeing the light of the day.

One of the areas in which Kerala mathematicians developed their skill was *Kuṭṭākāragaṇita.* It is a complex process dealing with rule of three continued factions and indeterminate equations. Though *Āryabhata* has given *Kuṭṭākāragaṇita* (*Āryabhatīya, Gaṇitapāda* 32, 33), it was developed and applied admirably to several problems by Kerala astronomers. One of the earliest accounts is given in Govindasrāmin's (800 - 850 A.D.) *Govindakṛti.* The work is not available now, but 22 stanzas describing the process have been quoted in the commentary of Saṅkaranārayaṇa (825 - 900 A.D.) on *Laghubhāskarīya.* This study of *Kuṭṭākāra* was followed by later mathematicians as evidenced by *Tantrasaṅgraha, Yuktibhāṣa* etc.

It is to be observed that the process of *Kuṭṭākāra* is motivated by Astronomy. Let x be the number of days, (round the earth), b the number of revolutions in a days. Then $\frac{bx}{a}$ gives the mean position of the planet. This may not be an integer.

Thus we can write $\frac{bx \pm c}{a} = y$ where x and y are integers. This takes the form $bx \sim ay = c$. c is called *kṣepa* or *śuddhi* according as it is positive or negative. Normally b is the number of revolutions and c may be in *rāśi* (30ś), degrees or minutes. b has to be multiplied by 12′, 360° or 21600 according as c in *rāśis*, degrees or minutes. This gives b and c in the same units (*rāśi*, degrees or minutes) whereas x and y are positive integers. We should first take $b > a$. We have to solve the equation.

$$bx - ay = c$$

or equivalently the equation $\frac{bx \pm c}{a} = y$. Here b is called the *bhājya* and a the *hāraka*. This is called *niragra Kuṭṭākāra*, the other type being *sāgra Kuṭṭākāra* discussed later. The equation $bx - ay = c$ can always be written in the form in which a and b are prime to each other. Even if they are not so they can be divided by the Highest Common Factor of a and b and reduced to lowest terms. This process is called *apavartana*. In this form they are called *dṛḍha* (firm) *bhājya* and *dṛḍha hāraka*. If $a = a'h$ and $b = b'h$, then we get $b'hx - a'hy = c/h$ and since $b'hx$ and $a'hy$ are integers, c/h is also an integer. We shall now consider equations of the form $bx - ay = c$ where a and b are prime to each other, where $a > 0$, $b > 0$ and c is a positive or negative integer,

Solution of the equation

Let $b > a$. Then divide b and a mutually and get the quotients q_1, q_2, . . . and remainders r_1, r_2, . . . Thus

$$b = q_1 a + r_1$$

$$a = q_2 r_1 + r_2$$

$$r_1 = q_3 r_2 + r_3$$

. . . .

. . . .

. . . .

$$r_{n-1} = q_{n+1} r_n + r_{n+1}$$

Continue till $r_{n+1} = 1$

For example, if

$$a = 449 \text{ and } b = 12372$$

then the quotients are 27, 1, 1, 4, 12 and remainders are 249, 200, 49, 4, 1

1	449	12372	27
4	200	249	1
	4	49	12
		1	

The method of solution is described below

We form the following *vallī* showing quotients

q_1

q_2

.

.

.

.

q_n

q_{n+1}

where q_n is the n^{th} quotient. We note that $b > a$ and b and a are called *hāraka* and *bhājya*. The remainders of odd order are

generated by b and are called *hārakaśeṣas* and the remainders of even order are called *bhājyaśeṣas*. Usually r_n is chosen as a *bhājyaśeṣa*. When *bhājyaśeṣa* is sufficiently small we find m called *mati* such that

$$\frac{r_n m + c}{r_{n-1}}$$

is an integer. The result is called *mati phala*. m can be chosen to be the smallest integer such that

$$\frac{r_n m + c}{r_{n-1}}$$

is an integer. In general the aim is to get the least integral values of x and y such that $x < a$ and $y < b$. Even if the value of m is not chosen like that, the required solution can be obtained by the process called *takṣaṇam*, described later. After finding m we form the *vallī*

$$\begin{array}{c} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_n \\ m \\ p \end{array}$$

where $p = \dfrac{r_n m + c}{r_{n-1}}$. Find $mq_n + p = s_1$ and write it against q_n. Then find

$$s_2 = q_{n-1} s_1 + m$$

$$s_3 = q_{n-2} s_2 + s_1$$

. . . .

Proceeding thus we complete the *vallī*, the process called *Vallyupasamhāra*. The last value is that of y and the next is that of x. In the process of solving the equation

$12372x - 449y = 1$, we mutually divide 12372 by 449 and get quotients

27, 1, 1, 4, 12,

and the remainders.

247, 200, 49, 4, 1.

We form the *vallī*

27
1
1
4
12
1

We can write the equation as

$$x = \frac{1 + 449y}{12372}.$$

449 is *bhājya* and the remainders of even order are *bhājyaseṣas*. We can choose 4, being sufficiently small, we observe that $\frac{4.12+1}{49} = 1$, 12 being the smallest integer to make the quotient an integer. Then $m = 12$. We form the *vallyupasamhāra*

27	3031
1	110
1	61
4	49
12	
1	

$$4.12 + 1 = 49$$
$$49.1 + 12 = 61$$
$$61.1 + 49 = 110$$
$$110.27 + 61 = 3031$$

We get $x = 110$ and $y = 3031$ clearly

$$12372.110 - 449.3031 = 1$$

and $110 < 449$ and $3031 < 12372$

The above problem can be done by taking a *hārakaśeṣa* instead of *bhājyaśeṣa*. Remainders of odd order are *hārakaśeṣa*. In this case *śuddhi* and *kṣepa* have to be interchanged. In the problem we have *kṣepa* equal to 1. We take *śuddhi* to be 1 and find *mati*. Thus *mati* is m such that $\frac{49m-1}{200}$ is an integer. We can take $m = 49$. Then

$$\frac{49.49-1}{200} = \frac{2401-1}{200} = 12$$

Matiphala = 12. We form the *vallī*

27	3031
1	110
1	61
49	
12	

$$49.1 + 12 = 61$$

$$61.1 + 49 = 110$$

$$110.27 + 61 = 3031$$

We get the same solution.

<u>Other cases</u>

We can always write equation in the form $bx - ay = c$ where $b > a$ where $c > 0$. $x = \frac{ay+c}{b}$ where a is called the *bhājya* and b *hāraka*. We considered above the case $b > a$. We shall discuss the other case now i.e., $b > a$.

In this case also a and b have to be mutually divided as before and the quotients and remainders obtained. In this case the *bhājyaseṣas* are of odd order. Consider the example

$$449y - 12372x = 1$$

i.e. $$\frac{1+12372x}{449} = y$$

The quotients are 27, 1, 1, 4, 12, and remainders are 249, 200, 49, 4, 1. We shall consider the 5^{th} remainder which is *bhājyaśeṣa*. The *mati*, m is to be chosen in order to make $\frac{1.m+1}{4}$ an integer. We can take $m = 3$. *Matiphala* is $\frac{3.1+1}{4} = 1$

We form the *vallī*

27	9341
1	339
1	188
4	151
12	37
3	
1	

$$12.3 + 1 = 37$$
$$37.4 + 12 = 151$$
$$151.1 + 37 = 188$$
$$188.1 + 151 = 339$$
$$27.339 + 188 = 9341$$
$$x = 339 \text{ and } y = 9341$$

One can verify that

$$449.9341 - 12372.339$$
$$= 4194109 - 4194108$$
$$= 1$$

The same problem can be done choosing a *hārakaśeṣa* instead of a *bhājyaśeṣa*. But the *mati m* has to be chosen such that $\frac{r_n m + c}{r_{n-1}}$ is an integer. If we choose the remainder 12 of even order, then choose m so that $\frac{12m-1}{4}$ is an integer. The rest is as before.

The process of *takṣaṇa*

Now it is necessary to describe the process called *takṣaṇa* in which the values are replaced by the remainders when solving the equation

$$bx - ay = c,$$

we look for the least integral values of x and y. When the values do not conform to this requirement, we do *takṣaṇam.* Let $x = x_1$ $y = y_1$ be solutions such that $x_1 > a$ and $y_1 > b$. If x and y are solutions then $bx - ay = c$. If any other solution is $x = x_1$, $y = y_1$, then $bx_1 - ay_1 = c$

Thus $b(x - x_1) - a(y - y_1) = 0$

$$\frac{x-x_1}{a} - \frac{y-y_1}{b} = p$$

We can write $\quad x = x_1 + ap$

$$y = y_1 + bp$$

In particular we can take $x = r_1 + ap$ and $y = r_2 + ap$. If $x < a$, then

$$ay = bx - c \text{ (where } c > 0)$$

$$< ab$$

i.e., $\quad y < b$

Therefore there exists solutions of the form (r_1, r_2). If now x_1 and y_1 are solutions we can write $x_1 = pa + r_1$ and $y_1 = pb + r_2$ where $0 \leq r_1 < a$ and $0 \leq r_2 < b$.

Then $\quad b(pa + r_1) - a(pb + r_2) = c$

i.e., $\quad bpa + br_1 - apb - ar_2 = c$

i.e., $\quad br_1 = ar_2 = c$

The solutions are given by $x = r_1$, and $y = r_2$ as desired.

Example

Solve $13x - 5y = 3$.

We divide 13 and 5 mutually and get the quotients 2, 1, 1 and remainders 3, 2,1. The equation is of the form $\frac{5y+3}{13} = x$ and *bhājya* is less than *hāraka*. Taking the remainder 1 of odd order, that is *hārakaseṣa*, we get *mati* m so as to get $\frac{1.m-3}{2}$ = an integer. We take $m = 9$ so that $\frac{9-3}{2} = 3$. *Matiphala* is 3. We form the *vallī*

r	q		
3	2	54	2
2	1	21	1
1	1	12	0
	9		
	3		

$9.1 + 3 = 12,\ 12.1 + 9 = 21,\ 21.2 + 12 = 54.$

The solution is given by $x = 21$ and $y = 54$. Since they are not the least, dividing by 5 and 13 respectively we get $x = 1$ and $y = 2$, the remainders as the required solution. Clearly

$$13.1 - 5.2 = 3$$

Takṣaṇam can be done at a lower stage. Since $12 > 1$ (the corresponding remainder), dividing 12 by 1 we get remainder 0. Since $21 > 2$, dividing 21 by 2 we get the remainder 1. Now $1.2 + 0 = 2$ is the value of y. Thus the solution is given by $x = 1$ and $y = 2$.

Procedure when it is difficult to find the *mati*

If is difficult to find the *mati* after a few steps, continue till remainder is 1 for *bhājya* or *hāraka*. In the former, we can take *kṣepa* as the *mati*. In either case *matiphala* is zero.

One can constitute the *vallī* as usual and find the solution. This means that when *bhājyahāraka* is 1, for the equation $y = \frac{ax+c}{b}$, $-cn'$ is the *mati* and *hārakaśeṣa* is 1. c is the *mati*.

Example

Consider the equation $13x - 5y = 3$

The quotients are 2, 1, 1 and remainders are 3, 2, 1. Take *mati* $= 3$ and *matiphala* $= 0$ and construct the *vallī*

q	
2	15
1	6
1	3
3	
0	

Dividing b by 5 we get the remainder = 1 and dividing a by 15 we get the remainder 2, the solution is given by $x = 1$ and $y = 2$.

$$13x - 5y = 3$$

In this we can write

$$\frac{13x-3}{5} = y$$

so that 13 is *bhājya* and since we get -3 (*śuddhi*) as the constant, $+3$ is *mati.*

We shall next consider the case

$$5y - 13x = 3$$

In this case $\dfrac{13x+3}{5} = y$. Therefore *mati* is -3. One can take *mati* as -3 and proceed. Then we get the *vallī*

2	-5
1	-6
1	-3
-3	
0	

The solutions are given by $x = -6$ and $y = -15$. Effecting *takṣaṇam* we get

$$x = -6 + 5.2 = -6 + 10 = 4$$

$$y = -15 + 2.13 = 11$$

$$5.11 - 13.4 = 55 - 52 = 3$$

In general the negative numbers are not used. The *mati* is taken to be positive and the solution obtained in the usual way if the solution is x_1, and y_1, then $b - x_1$ and $a - y_1$ are taken as the solutions.

APPENDIX III

MATHEMATICAL JUSTIFICATION FOR THE PROCEDURE OF EXTRACTING SQUARE, CUBE ROOTS GIVEN IN CHAPTER I, VERSE 19

by Dr. V.K. Krishnan

1. Finding Square Roots

The procedure given in the verse can be stated in modern mathematical language as follows :

THE PROCEDURE : Let N be a positive integer. Starting with an arbitrary positive integer a_0, find a_n, b_n for $n = 1,2,3 \ldots$ as follows :

$$b_n = \left[\frac{N}{a_{n-1}}\right] , \quad a_n = \left[\frac{1}{2}(a_{n-1} + b_n)\right]$$

Then

(a) $a_n = a_{n+2}$ for some n ;

(b) if $a_n = a_{n+2}$ for some n, *then* $a_n = \left[\frac{1}{2}(a_n + a_{n+1})\right] = \left[\sqrt{N}\right]$

Proof (a) As usual, we use $[x]$ to denote the greatest of the integers not exceeding x. Let $\left[\sqrt{N}\right] = m$. Since

$a_n = \frac{1}{2}(a_{n-1} + b_n)$ or $1/2\,(a_{n-1} + b_n - 1)$, we see that

$$a_n \leq \frac{1}{2}(a_{n-1} + b_n) \leq \frac{1}{2}\left(a_{n-1} + \frac{N}{a_{n-1}}\right) < \frac{1}{2}(a_{n-1} + b_{n+1}) \leq a_n + 1$$

Thus

$$a_n = \left[\frac{1}{2}\left(a_{n-1} + \frac{N}{a_{n-1}}\right)\right] \quad \text{for all } n \geq 1.$$

Moreover

$$a_n + 1 > \frac{1}{2}\left(a_{n-1} + \frac{N}{a_{n-1}}\right) \geq \sqrt{N} \geq m$$

and hence $a_n^2 + 2a_n + 1 > N \geq m^2$. It follows that $a_n \geq m$ and $a_n^2 + 2a_n \geq N$ for all $n \geq 1$.

If $a_n \geq m + 1$, then $\frac{N}{a_n} \leq \frac{N}{m+1} < m+1 \leq a_n$, and so $2a_{n+1} \leq a_n + \frac{N}{a_n} < 2a_n$. Thus, $a_{n+1} < a_n$ if $a_n > m$. So we cannot have $a_n > m$ for all $n \geq 1$. Since $a_n \geq m$ for all $n \geq 1$, we see that $a_n = m$ for some n.

Let $a_n = m$. If $N = m^2 + 2m$, then

$$a_n + \frac{N}{a_n} = 2m + 2, \quad a_{n+1} = m+1$$

and hence, $a_{n+2} = m = a_n$. If $N < m^2 + 2m$, then

$$m^2 \leq N < m^2 + 2m, \quad 2m \leq a_n + \frac{N}{a_n} < 2m + 2,$$

and hence, $a_{n+1} = m = a_n$. This proves (a).

(b) Suppose that $a_n = a_{n+2}$ for some n. If $a_n \geq m + 1$ and $a_{n+1} \geq m + 1$, then $a_n > a_{n+1} > a_{n+2}$, as observed above. So we must have $a_n = m$ or $a_{n+1} = m$. Let $a_n = m$. Then $a_{n+1} = m$ or $m + 1$, as above, and $\left[\frac{1}{2}(a_n + a_{n+1})\right] = m$ in either case. If $a_{n+1} = m$, then the same argument shows that $\left[\frac{1}{2}(a_{n+1} + a_{n+2})\right] = m$ But $a_{n+1} + a_{n+2} = a_n + a_{n+1}$. This proves (b).

2. Finding Cube Roots

The procedure in this case is as follows in modern mathematics language.

The Procedure : Let N be a positive integer greater than 63. Starting with an arbitrary positive integer a_0, find a_n, b_n, c_n for $n = 1,2,3, \ldots$ as follows :

$$b_n = \left[\frac{N}{a_{n-1}}\right], \quad c_n = \left[\frac{b_n}{a_{n-1}}\right], \quad a_n = \left[\frac{1}{2}(a_{n-1} + c_n)\right].$$

Then

(a) $a_n = a_{n+2}$ for some n ;

(b) if $a_n = a_{n+2}$ for some n, then $\left[\frac{1}{2}(a_n + a_{n+1})\right] = \left[\sqrt[3]{N}\right]$.

Proof (a) As before $[x]$ denotes the integral part of x. Let $\sqrt[3]{N} = p$ and $[p] = m$. Note that $c_n = \left[\frac{N}{a_{n-1}^2}\right]$, and hence,

$$a_n \leq \frac{1}{2}(a_{n-1} + c_n) \leq \frac{1}{2}\left(a_{n-1} + \frac{N}{a_{n-1}^2}\right) < \frac{1}{2}(a_{n-1} + c_n + 1) \leq a_{n-1} + 1.$$

Let $f(x) = \frac{1}{2}\left(a_n + \frac{N}{x^2}\right), x > 0$. Then $a_n = [f\ (a_{n-1})]$ for $n \geq 1$. Moreover if $x \geq m+1$, then $N < x^3$, and so $f(x) < x$. This shows that $a_{n+1} < a_n$ if $a_n \geq m+1$. Consequently, we must have $a_n \leq m$ for some $n \geq 1$.

Case I : Let $a_n = m$ for some n. If $N < m^3 + 2m^2$, then

$m \leq \frac{N}{m^2} < m+2$, and so $m \leq f(m) < m+1$,

$a_{n+1} = [f(a_n)] = [f(m)] = m = a_n$.

If $N \geq m^3 + 2m^2$, then

$$m+2 \leq \frac{N}{m^2} \leq m+3 + \frac{3}{m} < m+4 ,$$

$$m-1 < \frac{N}{(m+1)^2} < m+1.$$

It follows that

$m+1 \leq f(m) < m+2, \quad m < f(m+1) < m+1.$

Thus, if $N \geq m^3 + 2m^2$, then

$a_{n+1} = [f(m)] = m+1, \quad a_{n+2} = [f(m+1)] = m = a_n.$

Moreover, $[\frac{1}{2}(a_n + a_{n+1})] = m$ whether $a_{n+1} = m$ or $m+1$.

Case II : Let $a_n = m-1$ for some n. Then $(a_n + 1)^3 \leq N < (a_n + 2)^3$. Since $a_n = m-1 \geq 3$, we get

$$a_n + 3 < \frac{N}{a_n^2} < a_n + 6 + \frac{12}{a_n} + \frac{8}{a_n^2} < a_n + 6 + 5,$$

$$a_n + \frac{3}{2} \leq f(a_n) < a_n + \frac{11}{2} .$$

Thus, $m + \frac{1}{2} \leq f(a_n) < m + 9/2$, and hence, $a_{n+1} = [f(a_n)] = m + k$, where $0 \leq k \leq 4$.

If $a_{n+1} = m$, then $a_{n+3} = a_{n+1}$ by Case I. Suppose that $a_{n+1} = m + 1$. Since

$$(m-2)(m+1)^2 < m^3 \leq N < (m+1)^3,$$

we see that $m - 2 < \dfrac{N}{(m+1)^2} < m + 1$, and hence,

$$m - \tfrac{1}{2} < f(m+1) < m + 1, \; a_{n+2} = [f(a_{n+1})] = m - 1 \text{ or } m.$$

If $a_{n+2} = m - 1$, then $a_{n+2} = a_n$, and if $a_{n+2} = m$, then apply Case I.

Next suppose that $a_{n+1} = m + k$, with $k = 2$, 3 or 4. Then

$$m + k \leq \frac{1}{2}\left(m - 1 + \frac{N}{(m-1)^2}\right) < m + k + 1 \; ,$$

$$m + 2k + 1 \leq \frac{N}{(m-1)^2} < m + 2k + 3 \; .$$

If $k = 2$, this implies that $(m+5)(m-1)^2 \leq N < (m+7)(m-1)^2$, and hence,

$$(m-2)\;(m+2)^2 < (m+5)(m-1)^2 \leq N < m(m+2)^2,$$

$$m < f(m+2) < m + 1.$$

It follows that if $a_{n+1} = m + 2$, then $a_{n+2} = m$. Similarly, we can see that if $k = 3$ or 4, then again, $a_{n+2} = m$. We can apply Case I if $a_{n+2} = m$.

Note that $a_n = a_{n+2}$ only when $a_{n+1} = m + 1$, and so $[\frac{1}{2}(a_n + a_{n+1})] = m$.

Case III : Let $a_n \le m-2$ for some $n \ge 1$. Then $a_n = p - s$ where $s \ge 2$.

Since $2f(x) = \frac{x}{2} + \frac{x}{2} + \frac{N}{x^2} \ge 3\left(\frac{N}{4}\right)^{\frac{1}{3}} p$ by AM-GM inequality, we get

$$f(x) > \frac{3}{2}\left(\frac{1}{4}\right)^{\frac{1}{3}} p > \frac{3}{2}\,\frac{5}{2} p \quad \text{for all } x \ge 0.$$

Thus, we see that $a_n + 1 > f(a_{n-1}) > (15/16)p$ for all $n \ge 1$. Hence $p - 1 \ge p - s + 1 > (15/16)p$. This implies that

$$p > 16,\ a_n > (14/16)p,\ s = p - a_n < p/8.$$

Since $a_n < p$ we have $3a_n^2 s < p^3 - a_n^3 < 3p^2 s$. Thus,

$$3s > \frac{N}{a_n^2} - a_n < 3\left(\frac{p}{a_n}\right)^2 s < 3\left(\frac{8}{7}\right)^2 s < 4s,$$

$$2a_n + 3s < 2f(a_n) < 2a_n + 4s.$$

Thus, $a_{n+1} + 1 > f(a_n) > a_n + 3s/2 = p + s/2$, and $a_{n+1} \le f(a_n) < a_n + 2s = p + s$. Let $a_{n+1} = p + t$. Then $t > s/2 - 1 > 0$ and $t < s$.

Similarly, since, $a_{n+1} > p$, we have $3p^2 t < a^3_{n+1} - p^3 < 3a^2_{n+1} t$. Thus

$$3\left(\frac{p}{a_{n+1}}\right)^2 t < a_{n+1} - \frac{N}{a_{n+1}^2} < 3t$$

Since $t < s < p/8$, we get $a_{n+1} < (9/8)p$, and so

$$3t > a_{n+1} - \frac{N}{a_{n+1}^2} > 3(8/9)^2 t > 2t,$$

$$3t - 2a_{n+1} > -2f(a_{n+1}) > 2t - 2a_{n+1};$$

that is, $t - 2p > -2f(a_{n+1}) > -2p$. Hence,

$$p - t/2 < f(a_{n+1}) < a_{n+2} + 1, \quad a_{n+2} \leq f(a_{n+1}) < p,$$

$$a_{n+2} > p - t/2 - 1 > p - s, = a_n, \quad a_{n+2} \leq m.$$

Thus, $a_n < a_{n+2} \leq m$. If $a_{n+2} \leq m - 2$, repeat the argument. Thus, we can find some $k > n$ with $a_k = m$ or $m - 1$, and then apply Case I or II. This finishes the proof of (a).

(b) Suppose that $a_n = a_{n+2}$ for some n. Then $a_{n+1} = [f(a_n)] = [f(a_{n+2})] = a_{n+3}$. It follows from Case III that $a_n > m - 2$ and $a_{n+1} > m - 2$. If $a_n \geq m + 1$ and $a_{n+1} \geq m + 1$, then $a_n > a_{n+1} > a_{n+2}$ as observed in the beginning of the proof. Thus, we get $a_n \leq m$ or $a_{n+1} \leq m$. If $a_n \leq m$, then $a_n = m$ or $m - 1$. In either case, we get $\left[\frac{1}{2}(a_n + a_{n+1})\right] = m$, as shown in Cases I and II. Similarly, if $a_{n+1} \leq m$, then $\left[\frac{1}{2}(a_{n+1} + a_{n+2})\right] = m$ since $a_{n+1} = a_{n+3}$. But $a_n + a_{n+1} = a_{n+1} + a_{n+2}$. This proves (b).

TECHNICAL TERMS

[φ - latitude of the place, δ - declination of the Sun, ω - the obliquity]

Adhimāsa	:	Intercalary Month
Agra	:	Amplitude at rising, North-West distance of the rising point from the East-West line or *R* Sine of that.
Ahargaṇa (*Dyugaṇa*)	:	Number of days from the epoch.
Ahorātra vṛtta	:	Diurnal Circle.
Ākāśa kakṣya	:	Boundary circle of the sky having linear distance travelled by a planet in *yuga*, equal to 124,74,72,00,76,000 *yojanas*.
Ākṣa	:	$\frac{Trijy\bar{a} \times \tan\theta}{\tan \omega}$
Akṣa	:	Terrestrial latitude.
Akṣa-dṛk-karma	:	Reduction due to the latitude.
Aksajyā	:	*R* sine (terrestrial latitude).
Ākṣavalana	:	Deflection due to latitude.
Aṅgula	:	A linear measure, inch.
Antyakrānti	:	Maximum declination (of the Sun). Taken as 24^0 or more accurately as $23^0 27'$
Apakrama	:	Declination, Obliquity of the ecliptic.
Apamabāṇa	:	*R*–*R* cos (declination).
Apamaṇḍala	:	Ecliptic.
Apavartana	:	Process of finding HCF, Abrader.
Ardhajyā (*jyārdha*)	:	*R* sine.
Arkāgra	:	The amplitude of the Sun at rising or the *R* sine of that.

Asu	:	Unit of time equal to 1/6 of a *vinādi* i.e. 4 seconds.
Asita	:	Non-illuminated part of the Moon.
Astalagna	:	Setting point of the ecliptic, point of intersection of the Western horizon and the ecliptic.
Astamaya	:	Setting, diurnal or heliacal.
Avama	:	Omitted lunar day.
Ayana	:	Northward or Southward motion called *Dakṣiṇāyana* or *Uttarāyana.*
Āyana-Calana	:	Precision of the equinoxes.
Āyana-dṛk-karma	:	Reduction for observation on the ecliptic.
Ayanāmśa	:	The angular distance between the First Point of sidereal zodiac and the First Point of Aries.
Bhacakra	:	Circle of asterisms.
Bhāga	:	Degree.
Bhagola	:	Sphere of asterisms, Zodiacal sphere.
Bhakūta	:	The two apexes of the circle cutting at right angles.
Bhoga	:	Daily motion.
Bhū-bhramaṇa	:	The rotation of the Earth.
Bhūcchāyā	:	Earth's shadow.
Bhūdina	:	Number of Terrestrial days since epoch. (No. of Civil days reckoned from sunrise to sunrise)
Bhuja (*Bhujā*)	:	Lateral side of a right angled triangle. In odd quadrant is the arc covered and in even quadrant is the part of the arc yet to be covered.
Bhujā-phala	:	Equation of centre. Correction for the non-uniform motion of the planet in the circular orbit.

Bhujajyā	:	*R* sine.
Bhujāntara-phala	:	Correction for the equation of time due to the eccentricity of the ecliptic.
Bhukti	:	Motion.
Bhūparidhi	:	Circumference of the Earth.
Cakra	:	Circle, Cycle, Zodiac.
Cakraliptā	:	Minutes contained in the circle (21, 600)
Cāndra	:	Lunar.
Cāndra Māsa	:	A lunation, the period from one New Moon to the next or Full Moon to the next.
Candragrahaṇa	:	Lunar eclipse.
Cāpa	:	Arc.
Cara	:	Motion.
Caradala, Carārdha	:	Half *cara.*
Carajyā	:	*R* sine (ascensional difference) = *R* tan θ tan δ, where θ is the latitude of the place and δ the declination of the Sun.
Caraprāṇa	:	*Prāṇa* or *asus* of ascensional difference.
Caturaśra	:	Quadrilateral.
Chādaka	:	Eclipsing body.
Chādya	:	Eclipsed body.
Chāyā	:	Shadow. *R* sine of the zenith distance (*Mahācchāyā*).
Cheda	:	Denominator.
Dakṣiṇottara rekhā, *Yamyottara rekhā*	:	North-South line, Meridian, Solsticial Colure.
Dakṣiṇottara vṛtta (*Maṇḍala*) *Yamyottara vṛtta* (*Maṇḍala*)	:	Celestial Meridian.

Dala	:	Half.
Deśāntara	:	Difference in terrestrial longitude, correction for that longitude.
Deśāntara-kalā	:	Difference in time due to terrestrial longitude.
Dhana	:	Additive.
Dhanus	:	Arc.
Dhruvonnati	:	Altitude of the celestial pole.
Dhruva	:	1. Celestial pole. 2. Zero position of planet.
Dhruvanakṣatra, Dhruvatāra	:	Pole Star.
Dhruvavṛtta	:	Meridian circle.
Digagrā	:	North-South distance of the rising point from the East-West line.
Dik	:	Direction.
Dik-sūtra	:	Straight lines indicating directions.
Dina-bhukti	:	Daily motion.
Dṛḍha	:	Reduced to lowest terms (in *Kuṭṭākārakriyā*).
Dṛg-vṛtta	:	Vertical circle.
Dṛgjyā	:	*R* sine of zenith distance.
Dṛggati	:	Arc of the ecliptic measured from the central ecliptic point or *R* sine thereof, *R* sine altitude of nanogesimal.
Dṛk-karma	:	Reduction to observation.
Dṛk kṣepa	:	Ecliptic zenith distance. Zenith distance of the nanogesimal or *R* sine thereof.
Dṛk-kṣepajyā	:	*R* sine of *Dṛk-kṣepa*.
Dṛk-kṣepa-lagna	:	Nanogesimal (point on the ecliptic 90^0 behind *lagna*).

Dṛk-kṣepa-vṛtta (*maṇḍala*)	:	Vertical circle through the central ecliptic point.
Dyujyā	:	*R* cosine δ, being the radius of the diurnal path, a small circle parallel to the equator.
Dyuvṛtta	:	Diurnal path which is a small circle parallel to the equator.
Gaṇita	:	Mathematics, computation.
Gata	:	Elapsed portion of days, *nāḍikas* etc.
Ghana	:	Cube.
Ghana mūla	:	Cube root
Ghana saṅkalita	:	Sum of a series of cubes of natural numbers.
Ghaṭikā	:	*Nāḍikā*, unit of time equal to 24 minutes.
Ghaṭikāvṛtta (*Maṇḍala*)	:	Celestial equator.
Gola	:	Sphere.
Graha	:	Planet, including the Sun, the Moon, the *Ucca* (apogee–aphelion) and *pāta*, ascending node.
Grahabhukti	:	Daily motion of a planet.
Grāhaka	:	Eclipsing body (shadow in lunar eclipse, the Moon in solar eclipse)
Grahaṇa	:	1. Occultation. 2. Eclipse.
Grahayoga	:	Conjunction of planets.
Grāhya	:	Eclipsed body.
Grāsa	:	Measure of eclipse. Submergence in eclipse.
Guṇa	:	*R* sine. Three as *bhūta saṅkyā.*
Guṇakāra	:	Multiplier.
Guṇana	:	Multiplication.
Guṇya	:	Multiplicand.

Gurvakṣara	:	The sixtieth part of a *vināḍi.*
Hārajyā	:	$\frac{\textit{Trijyā}\ (1-\cos\delta)}{\cos\varphi \cos\delta}$
Icchā	:	One of the three quantities in rule of three.
Icchā-phala	:	Result corresponding to *Icchā.*
Iṣṭa	:	Desired (number, quantity etc.)
Jyā	:	*R* sine.
Jūka	:	*Tulā.*
Jyā-khaṇḍa	:	*R* sine segment. *R* sine difference.
Jyārdha	:	*R* sine.
Jyotiścakra	:	Circle of asterisms.
Kakṣyā	:	Orbit.
Kakṣyā-pratimaṇḍala	:	Eccentric circle.
Kakṣyāvṛtta (*maṇḍala*)	:	Deferent, mean orbit.
Kalā	:	Minute of arc.
Kāla lagna	:	R.A.(Right Ascension) of the East point.
Kalyādi	:	Commencing from Kali.
Kālajyā (*Kālāmśa*)	:	Angle between two points of time in degrees.
Kalidina	:	Number of days elapsed since the commencement of Kali yuga.
Kaliyuga	:	The aeon which commenced on 18.2.3102 B.C. at sunrise at Laṅka
Kalyādidhruva	:	Zero positions of planets at the commencement of Kaliyuga.
Kapāla	:	Hemisphere.
Karaṇa	:	Half a *tithi.*
Karṇa	:	Hypotenuse.
Kendra	:	Anomaly, Centre of a circle.

Khagola	:	Sphere of the sky.
Khaṇḍa grahaṇa	:	Partial eclipse.
Khaṇḍajyā	:	*R* sine – segment.
Koṭi	:	Complement of *bhuja*. Vertical side of a right angled triangle.
Koṭijyā	:	*R* cosine *R* sine *koṭi* = *R* cosine *bhujā*.
Krānti	:	Declination.
Krānti maṇḍala	:	Zodiacal circle.
Krāntijyā	:	*R* sine (declination)
Kṛti	:	Square.
Kṣepa (*Vikṣepa*)	:	1. Celestial latitude. 2. Additional quantity.
Kṣetra	:	Geometrical figure.
Kṣetra phala	:	Area.
Kṣitija	:	Horizon.
Kṣitijyā	:	$R \tan \varphi \sin \delta$
Kuṭṭākāra	:	Inderminate equation, pulverizer
Lagna	:	Point of intersection of the ecliptic and the eastern horizon.
Lamba	:	1. Latitude 2. Co-latitude.
Lambajyā	:	*R* sine (co-latitude) = $R \cos \varphi$.
Lambana	:	1. $R \cos \varphi$. Parallax in longitude.
Lambana nāḍikā	:	Parallax in longitude in terms of *nāḍikās*.
Laṅkā	:	Point on the terrestrial equator corresponding to 0^0 longitude.
Lāṭa	:	A type of *vyatīpāta*, when the sum of *sāyana* longitudes of the Sun and the Moon is 180^0.
Liptā	:	Minute of arc.

Madhya	:	Mean.
Madhya gati	:	Mean motion.
Madhya graha	:	Mean planet.
Madhyama	:	Mean.
Mahācchāyā	:	Great shadow. The distance of the foot of the *Mahā śaṅku* to the centre of the Earth. R sine (zenith distance).
Mahā śaṅku	:	Great gnomon, the perpendicular dropped from the Sun to the earth's plane. *R* sine of altitude.
Manda	:	Slow.
Mandakarma	:	*Manda* correction in computation of planetary position.
Mandakarṇa	:	Hypotenuse associated with *Mandocca.*
Mandakendra	:	Mean anomaly, Mean longitude – longitude of *Mandocca.*
Mandaphala	:	*Manda* correction, Equation of centre.
Mandavṛtta,	:	Epicycle of the equation of centre.
Maṇḍala	:	A circle.
Mauḍhya	:	The invisibility of planet due to proximity of the Sun.
Mati	:	Optional number in *Kuṭṭākārakriyā* (see Appendix II)
Matsya	:	Figure of fish formed by two arcs.
Meṣādi	:	The first point of Aries.
Mokṣa	:	Emergence in the eclipse, last point of contact.
Nāḍikā	:	*Ghaṭikā, Nāḍī,* a measure of time equal to 24 minutes.
Nākṣatra kakṣyā	:	Orbit of the asterisms with circumference equal to 17, 32, 60, 008 *yojanas.* 60 times that of the Sun.

Nakṣatra varṣa	:	Sidereal year, the time for the Sun to move from *Meṣādi* to next *Meṣādi.* The duration is *Makuṭolbaṇakṛṣṇa tālaḥ* (365^d 15^n 31^v 15^g)
Nata	:	Zenith distance.
Natajyā	:	*R* sine of zenith distance.
Nati	:	Parallax in latitude.
Nicoccavṛtta (*Maṇḍala*)	:	Epicycle.
Nīmilana	:	Beginning of total eclipse.
Nirakṣa	:	Equator at which latitude is zero.
Oja	:	Odd. For example the first and third quadrants.
Pada	:	Square root.
Pakṣa	:	Bright or dark half of a lunation.
Palabhā	:	The length of the shadow of the gnomon at Midday on the equinoctial day. $h \tan \varphi$, where h is the height of the *śaṅku* and φ is the latitude of the place.
Palajyā	:	*R* sine of latitude.
Paramagrāsa	:	Maximum obscuration in an eclipse.
Paramakrānti	:	Maximum declination taken to be 24^0 or more accurately as $23^0\ 27'$.
Paramāpakrama	:	Maximum declination.
Paraśaṅku	:	*R* sine of greater altitude (*R* sine of meridian altitude.
Paridhi	:	Circumference.
Parilekhā	:	Diagrammatic representation.
Parva	:	End point of New or Full Moon.
Paryaya	:	Revolution of a planet.
Pāta	:	Node, ascending node. (Rāhu in the case of the Moon).

Phala	:	Result.
Prāglagna	:	Commonly called *Lagna.* The point of intersection of the Eastern horizon and the ecliptic.
Pramāṇa	:	First element of the proportional, antecedent. *Pramaṇa, pramaṇaphala, iccha* and *icchaphala* constitute the four elements of a proportional.
Prāṇa	:	One sixth of a *vināḍi* equal to four seconds.
Pratipat	:	First day of the lunar fortnight called *śukla pratipat* or *kṛṣṇa pratipat* according as the Moon waxes or wanes.
Pūrvāpara	:	Prime vertical.
Pūrvāpara rekhā	:	East-West line.
Pūrvavisuvat	:	Vernal equinox.
Rāhu (*Candra pāta*)	:	Ascending node of the Moon's orbit.
Rāśi	:	Sign; arc of 30^0.
Rāśi cakra	:	Ecliptic.
Ṛkṣa (*nakṣatra, bha, tāra*)	:	Asterism, Star.
Ṛṇa	:	Negative, subtractive.
Samamaṇḍala	:	Prime vertical.
Samamaṇḍala śaṅku	:	*R* sine of the altitude of the Sun on the prime vertical.
Samparkārdha	:	Half the sum of the angular diameters of the eclipsed and eclipsing bodies.
Samsarpa	:	A lunar month preceding a lunar month called *amhaspati.*
Śaṅkvagra	:	North-South distance of rising or setting point from the tip of the shadow. *Agrā* ± *natijyā.*

Saṅkrānti	:	Solar ingress, entrance of the Sun into the *rāsis Meṣa, Vṛṣbha* etc.
Śaṅku	:	1. Gnomon (is of 12 *aṅgulas* or 52 *aṅgulas*) 2. *Mahā Śaṅku* is the perpendicular dropped from the Sun to the Earth line or R sine (altitude).
Śaṅku koṭi	:	Complement of altitude i.e. zenith distance.
śara	:	Literally 'arrow', synonymous with *iṣu, bāṇa* etc. It is equal to $R - R$ sine (or $R + R$ sine at times).
Sārpamastaka	:	*Vyātīpāta* when the sum of the longitudes of the Sun and Moon is equal to $7^r\ 16^0\ 4'$.
Sāvanadina	:	Civil day, the duration between two successive sunrises.
Śīghra karma	:	The process of *śīghra* correction.
Śīghra karṇa	:	Hypotenuse related to *śīghra vṛtta*
Śīghra paridhi	:	Epicycle of the *śīghravṛtta* (i.e. equation of conjunction).
Śīghrocca	:	Higher apses of the epicycle related to *śīghra* correction.
Śiṣṭa	:	Remainder.
Sita	:	'White' (illuminated) part of the Moon, phase of the Moon.
Sphuṭa (*graha*)	:	True position of a planet. Longitude of a planet.
Sphuṭa gati	:	True daily motion of a planet.
Sphuṭa vikṣepa	:	Latitude corrected for parallax.
Sṛṅgonnati	:	Elevation of the Moon's horns.
Śruti	:	Hypotenuse.
Sthityardha	:	Half the duration of an eclipse.

Śūnya	:	Zero.
Sūrya grahaṇa	:	Solar eclipse.
Svadeśahāraka	:	$R \sec \varphi$
Svam	:	Positive, Additive.
Svastikā	:	The cardinal points, the Zenith and the Nadir are the six *svastikas* of the celestial sphere.
Tamas	:	Shadow cone of the earth at the Moon's distance. Moon's ascending node, viz. Rāhu.
Tārāgraha	:	Star planets viz. Mars, Mercury, Jupiter, Venus and Saturn.
Tithi	:	Lunar day. The first lunar day being the time taken by the Moon to trace 12^0 w.r.t. the Sun. Since New Moon, the second, the time to trace 12^0 to 24^0 and so on.
Tithikṣaya (Avama)	:	Omitted lunar day.
Tribhuja	:	Triangle.
Trijyā	:	R sine 90^0. The radius of length 3438' in the table of R sines.
Trirāśika	:	Rule of three.
Ucca	:	Higher apses relating to the epicycle.
Uccanīca vṛtta	:	Epicycle.
Udaya	:	Rising, diurnal or heliacal.
Udaya lagna	:	Rising point of the ecliptic, point of intersection of the Eastern horizon and the ecliptic, Ascendant.
Udayajyā	:	R sine amplitude of the rising point on the ecliptic.
Unmaṇḍala	:	Six O' clock circle. Equatorial horizon.
Unmīlana	:	End of the total eclipse.

Upāntya	:	Penultimate, Penultimate term.
Utkramajyā	:	R – versed sine, R – R cosine.
Uttara viṣuvat	:	Autumnal equinox.
Vaidhṛta	:	A kind of *Vyatipāta* which occurs when the sum of the *sāyana* longitudes of the Sun and the Moon is 360^0.
Vakra	:	Retrograde.
Valana	:	Deflection. It can be in latitude or declination.
Vallī	:	Literally 'creeper'. A series of results in *Kuṭṭākāragaṇita.*
Varga	:	Square.
Vargamūla	:	Square root.
Vikalā	:	Second of arc.
Vikṣepa	:	Latitude of Moon or other planets.
Vikṣepa maṇḍala (*Vimaṇḍala*)	:	Orbit of a planet.
Viliptā	:	Second of arc.
Vimardārdha	:	Half the duration in a total eclipse.
Vināḍī (*Vināḍīkā*)	:	One-sixtieth of a *nāḍika.*
Viṣama	:	Odd.
Viśeṣa	:	Difference.
Viṣkambha	:	1. Diameter. 2. The first of the 27 *yogas* formed by adding the longitudes of the Sun and the Moon.
Viṣuvacchāyā	:	Equinoctial shadow = $h \tan \varphi$, where h is the height of the *śaṅku* and φ the latitude of the place.
Viṣuvad vṛtta (*Maṇḍala*)	:	Celestial equator.
Viṣuvadbha (*Viṣuvacchāyā*)	:	Length of the shadow of the gnomon of 12 units on the equinoctial day at noon.

Viṣuvat	:	Equinox.
Viṣuvat karṇa	:	Hypotenuse of the equinoctial shadow.
Vivara	:	Difference.
Vṛtta	:	Circle.
Vṛtta kendra	:	Centre of a circle.
Vṛtta paridhi	:	Circumference of a circle.
Vyāsārdha	:	Radius.
Vyatīpāta	:	The time when the sum of the *sāyana* longitudes of the Sun and the Moon is 180^0 or 360^0 and the declinations are equal.
Vyavakalana	:	Subtraction.
Yamyottara vṛtta	:	Meridian.
Yāmya	:	Southern.
Yamyottara rekhā	:	South North line.
Yoga	:	1. Conjunction of planets. 2. *Nitya yoga* decided on the basis of the sum of the longitudes of the a Sun and the Moon (*Viṣkambha*, *Prīti* etc.)
Yojana	:	A unit of length equal to 4, 8 or 16 miles.
Yojana gati	:	Daily motion of planets in *yojanas.*
Yuga	:	Aeon, Kaliyuga is of 4, 32, 000 years and *Mahāyuga* consists of the four *yugas* - Kṛta, Tretā, Dvāpara and Kali, with durations, 4 times, 3 times, 2 times, and once of *Kali*, the total duration being 4, 320, 000 years.
Yugma	:	Even, couple.

TABLE 1

TABLE OF JYĀS

The following is the table of *R* sines for arcs from 0 to 90^0 for intervals of 3^0 45′.

	Arc	Āryabhaṭa's Value	Madhava's Value
1	225′	225′	224′ 50″ 22‴
2	450′	449′	448′ 42″ 58‴
3	675′	671′	670′ 40″ 11‴
4	900′	890′	889′ 45″ 15‴
5	1125′	1105′	1105′ 1″ 39‴
6	1350′	1315′	1315′ 34″ 7‴
7	1575′	1520′	1520′ 28″ 35‴
8	1800′	1719′	1718′ 52″ 24‴
9	2025′	1910′	1909′ 54″ 35‴
10	2250′	2093′	2092′ 46″ 3‴
11	2475′	2267′	2266′ 39″ 50‴
12	2700′	2431′	2430′ 51″ 15‴
13	2925′	2585′	2584′ 38″ 6‴
14	3150′	2728′	2727′ 20″ 52‴
15	3375′	2859′	2858′ 22″ 55‴
16	3600′	2978′	2977′ 10″ 34‴
17	3825′	3084′	3083′ 13″ 17‴
18	4050′	3177′	3176′ 3″ 50‴
19	4275′	3256′	3255′ 18″ 22‴
20	4500′	3321′	3320′ 36″ 30‴
21	4725′	3372′	3371′ 41″ 29‴
22	4950′	3409′	3408′ 20″ 11‴
23	5175′	3431′	3430′ 23″ 11‴
24	5400′	3438′	3437′ 44′ 48‴

TABLE 2*

PRĀṆA KALĀNTARAJYĀ

This gives the value of | R sin (ℓ – α) | where ℓ is the longitude and α is the right ascension of the Sun. In the first and third quadrants it is negative and in the second and fourth quadrants it is positive. It is given for the longitude of the Sun reduced to first quadrant and divisions of 3^0 45′. The intermediate values can be got by interpolation. They are in minutes of angle.

1.	*Dhanyona*	19
2.	*Saṅgona*	37
3.	*Samena*	57
4.	*Rāsanam*	72
5.	*Jahāna*	88
6.	*Ratnasya*	102
7.	*Śukasya*	115
8.	*Candrika*	126
9.	*Ṣaḍhāsya*	136
10.	*Gūḍhasya*	143
11.	*Suvākya*	147
12.	*Dhāvakam*	149
13.	*Dhavopi*	149
14.	*Tanvīḍya*	146
15.	*Nivārya*	140
16.	*Rāgakṛt*	132
17.	*Putrasya*	121
18.	*Dhīnamya*	109
19.	*Vidhāna*	94
20.	*Sarthanam*	77
21.	*Dharmonu*	59
22.	*Nirbhinna*	40
23.	*Niranna*	20
24.	*Nānanah*	0

★ All *Vākyas* found in Table 2 - Table 7 are from *Pañcabodha.*

TABLE 3
GUḌHĀMENAKADI JYĀ

These are the *jyās* of arcs which exceed the *jyās* by 1″, 2″ . . . , 24″.

1.	*Guḍhāmenakā*	105′ 43″
2.	*Pujyo gaṅgeyaḥ*	133′ 11″
3.	*Candraḥ śrīmayaḥ*	152′ 26″
4.	*Stambhaḥ sthitikṛt*	167′ 46″
5.	*Guḍhohni dīpaḥ*	180′ 43″
6.	*Prājño radheyaḥ*	192′ 2″
7.	*Dhanī trinetraḥ*	202′ 9″
8.	*Ugraḥ kukkuraḥ*	211′ 20″
9.	*Sātvadhī puraḥ*	219′ 47″
10.	*Svargaḥ suraṣtram*	227′ 34″
11.	*Himavan gauraḥ*	234′ 58″
12.	*Ramoyam vīrah*	241′ 52″
13.	*Vārijam bhadram*	248′ 24″
14.	*Tāṇḍavam miśram*	254′ 36″
15.	*Kulau nācāraḥ*	260′ 31″
16.	*Ājyāptiḥ kṣīrāt*	266′ 10″
17.	*Caṇḍaḥ kesari*	271′ 36″
18.	*Dhāvati sarit*	276′ 49″
19.	*Umeṣto hāraḥ*	281′ 50″
20.	*Adbhuto hāraḥ*	286′ 40″
21.	*Krūra yoddharaḥ*	291′ 22″
22.	*Śiśurmadhuraḥ*	295′ 55″
23.	*Dhairyam jñanāṅgam*	300′ 19″
24.	*Tilaugho nīlaḥ*	304′ 36″

TABLE 4

KRĀNTIJYĀ

TABLE OF DECLINATION OF THE SUN

The following table gives the values of $R \sin \delta$, where δ is the declination of the Sun, when the *sāyana* longitude is known. The values are given for intervals of $3^0\, 45'$. It is positive when longitude is *Meṣādi* and negative ehen it is *Tulādi.* They are in minutes of angle.

1.	*Yuddenu*	91
2.	*Rajanya*	182
3.	*Lasendra*	273
4.	*Rantige*	362
5.	*Dhavābha*	449
6.	*Mārgeṇa*	535
7.	*Jayanti*	618
8.	*Dhīdhṛtam*	699
9.	*Sartharthanam*	777
10.	*Kṛṣṇajanam*	851
11.	*Kharodhanam*	922
12.	*Dadurdhanam*	988
13.	*Kṛṣṇanṛpaḥ*	1051
14.	*Dhanī paṭuḥ*	1109
15.	*Gatasyayaḥ*	1163
16.	*Puṇyarayayaḥ*	1211
17.	*Vāśapriyaḥ*	1254
18.	*Radhāpriyaḥ*	1293
19.	*Bhadralayaḥ*	1324
20.	*Yamālayaḥ*	1351
21.	*Kathalayaḥ*	1371
22.	*Todālayaḥ*	1386
23.	*Mudhalayaḥ*	1395
24.	*Dugdhālayaḥ*	1398

For the Moon also this is found first and the sum or the difference of this and the latitude is taken as the declination (This is only approximate).

TABLE 5
CARAJYĀ FOR ĀLATHUR

Carajyā gives the accessional difference of any body in diurnal motion. It is given by sin *cara* = tan φ tan δ, where φ is the latitude of the place and δ is the declination. The following table gives the value of R tan φ tan δ for Ālathur in minutes of angle for divisions of $3^0\ 45'$, when δ is known. If p is the *palaṅgula* for Ālathur, and p' is the *palaṅgula* of another place then *carajyā* for the place = $\frac{\textit{carajyā} \text{ for Ālathur}}{\textit{palāṅgula} \text{ for Ālathur}} \times p'$

1.	*Lambana*	43
2.	*Sarjanam*	87
3.	*Pālaya*	131
4.	*Sārthakṛt*	177
5.	*Madrirāt*	225
6.	*Gosthira*	273
7.	*Mekhalā*	325
8.	*Nirjala*	380

TABLE 6
LUNAR JYĀS

These are the *jyas* of the *mandaphala* of the Moon given by $\frac{7}{80} \times (R \sin m)$ where *m* is the *manda kendra.* Since the angles are small the *jyas* and the arcs in minutes are nearly the same. These are called *naronvādijyās* in *Parahita* system referred to in the text.

1.	*naronu*	20
2.	*dhīgonu*	39
3.	*dhamena*	59
4.	*hāsanam*	78
5.	*sudhenu*	97
6.	*mānyasya*	115
7.	*balasya*	132
8.	*nāmakām*	150
9.	*sutasya*	167
10.	*lohasya*	183
11.	*jaḷasya*	198
12.	*gopuram*	213
13.	*cāritra*	226
14.	*dhīgotra*	239
15.	*nṛmātra*	250
16.	*pakṣhirāt*	261
17.	*asitra*	270
18.	*dāsitra*	278
19.	*mahendra*	285
20.	*kandharaḥ*	291
21.	*mṛdhāra*	295
22.	*dugdhagra*	298
23.	*nṛnāga*	300
24.	*pannagaḥ*	301

TABLE 7
JYĀS FOR SATURN

One can compute the positions of the Sun and the Moon using the *mandaparidhis* and *mandaphala* using the table of *jyās*. For others, *manda* and *śīghraphalas* can be found in the way prescribed and their positions can be computed. But tables are generally prepared for computation of the planetary positions. Instead of calculating the *manda* and *śīghraphalas* using the formulae, they can be obtained direct from the tables.

The *jyās* for Saturn are given as illustration. They are in minutes and actually give the arcs though known as *jyās*.

MANDAJYĀS OF SATURN

tīranut	26	*dhījagu*	389
gomanam	53	*kṣepavit*	416
prānjanam	82	*rāghavo*	442
prāpako	112	*bhūtavit*	464
gravako	142	*simhavat*	487
gāthako	173	*tanniśi*	506
menire	205	*śrīrāma*	522
tumburuḥ	236	*tadguṇā*	536
doṣarāt	268	*māgharṇiḥ*	545
dhidharo	299	*meśaśi*	555
nālago	330	*dharmiṇaḥ*	559
nītigo	360	*pakṣiṇaḥ*	561

ŚĪGHRAPHALA

MAKARĀDIJYĀS

tarujña	26	*dhavendra*	249
śaivajña	45	*rakṣāri*	262
śucijña	65	*cacāra*	266
dehinā	84	*sohari*	287
dhanāḍhya	109	*dadhatra*	298
dhīrādhya	129	*dhinnāga*	309
javaya	148	*sukīla*	317
sañcayaḥ	167	*bhūriguḥ*	324
madāḍhya	185	*kulāli*	331
ratnāgra	202	*śailāṅga*	335
dayāgra	218	*nabhoga*	340
bhogirāt	224	*garbhagaḥ*	343

KARKYĀDIJYĀS

harijña	28	*mahendra*	285
carmajña	56	*dhugdhāgra*	299
guhena	83	*dhanāṅgi*	309
dhenuko	109	*divyaguḥ*	318
viloki	124	*taraṇgi*	326
sāmarthya	157	*gaṅgāmbu*	333
nadīpa	180	*jalāla*	338
kinnaraḥ	201	*kumbhagi*	341
paratra	221	*bhavāṅgi*	344
dhīgotra	239	*śobhāṅgi*	345
kṣametra	256	*vibhaṅga*	344
kesarī	271	*lābhagi*	343

TABLE 8
FOOT VAKYAS

The following table gives the lengths of the shadows corresponding to the *nāḍikās* after sunrise or before sun set for the months *Meṣa, Vṛsabha* etc. The height of the gnomon is 52 *aṅgulas* and 8 *aṅgulas* make one foot. These are only approximate and can be applied only to low latitudes.

Nāḍikā	*Meṣa* or *Simha* f – a	*Kanyā* or *Mīna* f – a	*Tulā* or *Kumbha* f – a	*Vṛścika* or *Makara* f – a	*Dhanus* f – a	*Vṛsabha* or *Karkaṭaka* f – a	*Mithuna* f – a
1.	64 – 0	64 – 0	64 – 0	68 – 0	70 – 0	67 – 0	69 – 0
2.	32 – 0	32 – 0	32 – 0	34 – 0	35 – 0	33 – 0	34 – 0
3.	21 – 0	20 – 0	23 – 0	22 – 0	23 – 0	22 – 0	22 – 0
4.	15 – 0	15 – 0	15 – 0	16 – 0	17 – 0	16 – 0	16 – 0
5.	12 – 0	12 – 0	12 – 0	12 – 0	13 – 0	12 – 0	13 – 0
6.	9 – 3	9 – 1	9 – 5	10 – 8	11 – 0	9 – 5	10 – 0
7.	7 – 4	7 – 3	7 – 7	8 – 6	9 – 1	8 – 0	8 – 2
8.	6 – 1	6 – 1	6 – 5	7 – 3	7 – 6	6 – 5	6 – 7
9.	5 – 0	5 – 0	5 – 4	6 – 3	6 – 5	5 – 3	5 – 5
10.	4 – 0	4 – 0	4 – 5	5 – 4	5 – 6	4 – 3	4 – 5

Nāḍikā	*Meṣa* or *Simha* f – a	*Kanyā* or *Mīna* f – a	*Tulā* or *Kumbha* f – a	*Vṛścika* or *Makara* f – a	*Dhanus* f – a	*Vṛsabha* or *Karkaṭaka* f – a	*Mithuna* f – a
11.	3 – 1	3 – 1	3 – 7	4 – 7	5 – 3	3 – 4	3 – 7
12.	2 – 3	2 – 3	3 – 3	4 – 3	5 – 0	2 – 6	3 – 1
13.	1 – 5	1 – 6	2 – 7	4 – 1	4 – 5	2 – 1	2 – 4
14.	0 – 6	1 – 3	2 – 6	4 – 0	4 – 4	1 – 5	2 – 7
15.	0 – 3	1 – 1	2 – 5	3 – 7	1 – 3	1 – 2	1 – 6
16.	0 – 0	0 – 0	0 – 0	0 – 0	0 – 0	1 – 1	1 – 5

MANUSCRIPT OF SADRATNAMĀLĀ
TEXT WITH COMMENTARY
IN MALAYALAM SCRIPT

PALM LEAF MANUSCRIPT OF

SADRATNAMĀḶĀ

IN

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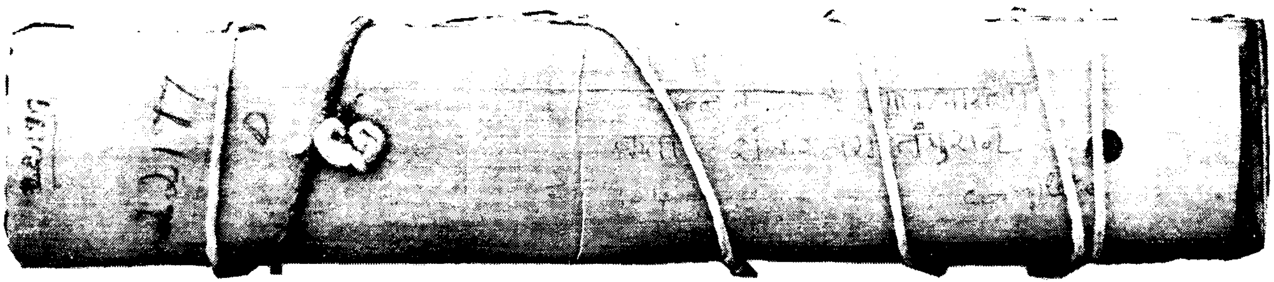

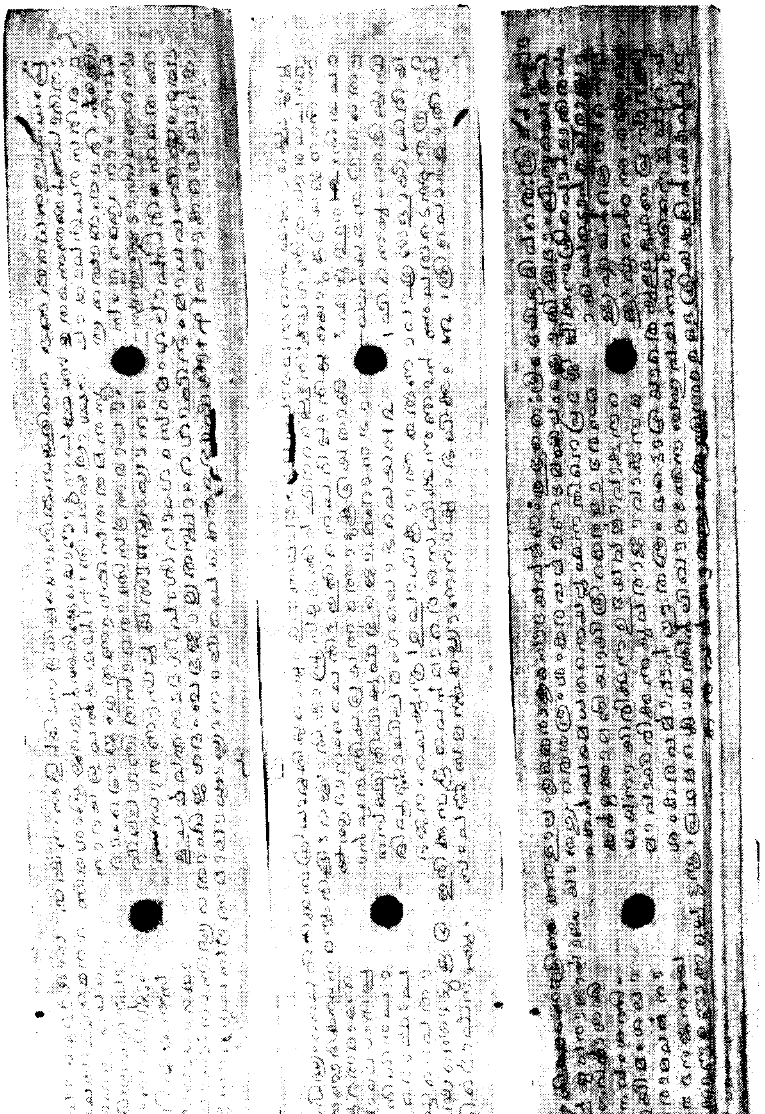

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VERSE INDEX

Note: Verses in the footnotes have been indicated against the relevant verses with the suffix 'fn'.

GENERAL INDEX

O

P

Indological Truths